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Hall effects on peristaltic flow of a Maxwell fluid in a porous medium

T. Hayat [a,*], N. Ali [a], S. Asghar [b]

[a] Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan
[b] Department of Mathematical Sciences, COMSATS Institute of Information Technology, Islamabad, Pakistan

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Abstract

This work is concerned with the peristaltic transport of an incompressible, electrically conducting Maxwell fluid in a planar channel. The flow in the porous space is due to a sinusoidal wave traveling on the channel walls. The Hall effect is taken into account and permeability of porous medium is considered uniform. Modified Darcy's law has been used to model the governing equation. An analytical solution is obtained, which satisfies the momentum equation for the case in which the amplitude ratio is small. The present theoretical model may be considered as mathematical representation to the case of gall bladder and bile duct with stones and dynamics of blood flow in living creatures. Finally, the graphical results are reported and discussed for various values of the physical parameters of interest.
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1. Introduction

Peristaltic transport is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of an extensible tube containing a liquid. It appears to be major mechanism for urine transport in ureter, food mixing and chyme movement in intestinal, transport of spermatozoa in cervical canal, transport of bile in bile ducts and so on. Technical roller and finger pumps also operate according to this rule.

To understand peristaltic action in different situations, several theoretical and experimental attempts have been made since the first investigation of Latham [1]. The literature on peristalsis is by now quite extensive. Important recent contributions to the topic include the works of Hayat et al. [2–4], Siddiqui et al. [5], Haroun [6], Elshahed and Haroun [7] and Siddiqui and Schwarz [8,9].

In all the above mentioned studies, no porous medium has been taken into account. But it is well known that flow through a porous medium has practical applications especially in geophysical fluid dynamics. Examples of natural porous media are beach sand, sandstone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. In the arterial systems of humans or animals, it is quite common to find localized narrowings, commonly caused by intravascular plaques. These stenosis disturb the normal pattern of blood flow through the artery. Acknowledge of flow characteristics in the vicinity of stenosis may help to further the understanding of some major complications which can arise such as, an ingrowth of tissue in the artery, the development of a coronary thrombosis, the weakening and bulging of the artery downstream from stenosis, etc. The investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. In some pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the lumen of coronary artery can be considered as equivalent to a porous medium. Recently, El Shehawey and Husseny [10] and El Shehawey et al. [11] studied the

* Corresponding author.
E-mail address: t_pensy@hotmail.com (T. Hayat).

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peristaltic mechanism of a Newtonian fluid through a porous medium.

To the best of the author's knowledge, no attempt is available in the literature which deals with the peristalsis of non-Newtonian fluid including the Hall effect through a porous medium. Even the study of peristaltic flow of an electrically conducting non-Newtonian fluid in a porous medium is not available. Also, the study of peristaltic flow of a Newtonian fluid through a porous medium with Hall effects has not been made yet. The present study fills the gap in these directions. The main objective of this Letter is to investigate the peristaltic flow of an electrically conducting Maxwell fluid in a channel through a porous medium with constant permeability. Modified Darcy's law for a Maxwell fluid including the Hall current has been used for the modeling. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron–atom-collision frequency, is high. This happens, when the magnetic field is high or when the collision frequency is low. In most cases, the Hall term has been ignored in applying Ohm's law as it has no marked effects for small and moderate values of the magnetic field. However, the current trend in the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable. Under these conditions, the Hall current is important and it has marked effects on the magnitude and direction of the current density and consequently on the magnetic-force term. Therefore, it is of interest to study the influence of the Hall current on the flow. The works dealing the influence of Hall current without peristalsis have been given in the studies [12–15]. The organization of the Letter is as follows.

We formulate the problem in Section 2. In Section 3, the analytic solution of the problem is provided for small amplitude ratio. Section 4 deals with the discussion. The conclusions have been provided in Section 5.

2. Flow analysis

We consider a two-dimensional channel of uniform thickness $2d$, filled with homogeneous electrically conducting Maxwell fluid through a porous medium. The fluid is taken incompressible and channel walls flexible. The flow is considered in the direction of x-axis and y-axis is taken normal to the flow. A uniform magnetic field with magnetic flux density vector $\mathbf{B} = (0, 0, \mathbf{B}_0)$ is applied (which is assumed to be the total magnetic field, as the induced magnetic field is neglected by taking a very small magnetic Reynolds number). The Hall effect is taken into account. The expression for the current density $\mathbf{J}$ including the Hall effect is given by

$$\mathbf{J} = \sigma\left[\mathbf{V} \times \mathbf{B}_0 - m(\mathbf{J} \times \mathbf{B}_0)\right] \tag{2.1}$$

in which σ is the electrical conductivity of the fluid, $\mathbf{V}$ is the velocity and m $(=\sigma B_0/en_e)$ is the Hall parameter, e is the electric charge and n_e is the number density of electrons. Eq. (2.1) may be solved in $\mathbf{J}$ to yield a Lorentz force vector in the form

$$\mathbf{J} \times \mathbf{B}_0 = -\frac{\sigma B_0^2 \theta}{\rho}\left[(u - mv)\hat{i} + (v + mu)\hat{j}\right], \tag{2.2}$$

where $\theta = 1/1 + m^2$, ρ is the fluid density, u and v are the x and y-components of the velocity and $\hat{i}$ and $\hat{j}$ are the unit vectors in x and y-directions, respectively.

The wall deformation due to the propagation of sinusoidal wave is

$$h(x, t) = \pm d \pm \eta, \quad \eta = a\cos\frac{2\pi}{\lambda}(x - ct), \tag{2.3}$$

in which λ is the wavelength, c is the wave speed, t is the time and a is the amplitude. In a porous medium the incompressible fluid motion is governed by the following equations

$$\operatorname{div}\mathbf{V} = 0, \tag{2.4}$$

$$\rho\frac{d\mathbf{V}}{dt} = -\nabla p + \operatorname{div}\mathbf{S} + \mathbf{J} \times \mathbf{B}_0 + \mathbf{r}. \tag{2.5}$$

In above equations d/dt is the material derivative, p is the hydrostatic pressure, $\mathbf{S}$ is the extra stress tensor and $\mathbf{r}$ is the Darcy's resistance for a fluid in a porous medium.

For a linear Maxwell fluid, the constitutive equation for the extra stress is

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\mathbf{S} = \mu\mathbf{A}, \tag{2.6}$$

where μ is the dynamic viscosity and τ is the relaxation time. The expression for $\mathbf{A}$ is defined as

$$\mathbf{A} = (\operatorname{grad}\mathbf{V}) + (\operatorname{grad}\mathbf{V})^T. \tag{2.7}$$

It is well known that in an unbounded porous medium, Darcy's law holds for a Newtonian fluid at low speed. This law provides a relation between pressure drop and velocity. According to this law the pressure drop induced by the frictional drag is directly proportional to the velocity. There are some studies [16,17] on Stoke's problem involving viscoelastic fluid through a porous medium. There is no study on peristalsis dealing viscoelastic fluid in a porous medium. On the basis of Oldroyd's model [18–23] the following law has been suggested

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\nabla p = -\frac{\mu\varphi}{k}(1 + \lambda_v)\mathbf{V}, \tag{2.8}$$

where k is the permeability, λ_v is the retardation time and φ is the porosity of the porous medium.

It is known that constitutive equation for Maxwell fluid can be obtained from the constitutive equation of an Oldroyd-B fluid by letting $\lambda_v = 0$. Since we have an interest in Maxwell fluid in this Letter, the filtration law for Maxwell fluid can be inferred from Eq. (2.8) as follows:

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\nabla p = -\frac{\mu\varphi}{k}\mathbf{V}. \tag{2.9}$$

Note that for $\tau = 0$, the above equation yields Darcy's law. Since the pressure gradient in Eq. (2.9) can also be interpreted as a measure of the resistance to flow in the bulk of the porous medium and $\mathbf{r}$ is a measure of the flow resistance offered by the solid matrix. Therefore $\mathbf{r}$ can be inferred from Eq. (2.9) to satisfy the following equation [24]

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\mathbf{r} = -\frac{\mu\varphi}{k}\mathbf{V}. \tag{2.10}$$

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