Physics Letters A 378 (2014) 2915–2923

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Tachyonic Cherenkov radiation in the absorptive aether

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ARTICLE INFO

Article history:
Received 21 July 2014
Received in revised form 10 August 2014
Accepted 11 August 2014
Available online 13 August 2014
Communicated by V.M. Agranovich

Keywords:
Tachyonic radiation fields
Complex permeability tensors and tachyon mass
Energy dissipation in a permeable spacetime
Green functions of dispersive Maxwell–Proca fields
Dissipative Cherenkov flux densities
Shock-heated plasma of supernova remnants

ABSTRACT

Dissipative tachyonic Cherenkov densities are derived and tested by performing a spectral fit to the γ-ray flux of supernova remnant (SNR) RX J1713.7 − 3946, measured over five frequency decades up to 100 TeV. The manifestly covariant formalism of tachyonic Maxwell–Proca radiation fields is developed in the spacetime aether, starting with the complex Lagrangian coupled to dispersive and dissipative permeability tensors. The spectral energy and flux densities of the radiation field are extracted by time averaging, the energy conservation law is derived, and the energy dissipation caused by the complex frequency-dependent permeabilities of the aether is quantified. The tachyonic mass-square in the field equations gives rise to transversally/longitudinally propagating flux components, with differing attenuation lengths determined by the imaginary part of the transversal/longitudinal dispersion relation. The spectral fit is performed with the classical tachyonic Cherenkov flux radiated by the shell-shocked electron plasma of SNR RX J1713.7 − 3946, exhibiting subexponential spectral decay.

1. Introduction

Decay is inherent in all composite matter, and the spacetime structure employed in physical modeling should reflect this fact. Here, we consider tachyonic wave propagation in a dissipative spacetime, the physical manifestation of the all-pervading aether. The aim is to quantify the effect of absorption on the spectral decay of radiation densities, to develop the general formalism and work out a specific example, dissipative tachyonic Cherenkov radiation [1–9], and to probe the decaying energy flux by performing a spectral fit to the γ-ray emission from an ultra-relativistic plasma. For the latter, we consider the shock-heated electron population of a supernova remnant.

To this end, we start with the complex Lagrangian of a Maxwell–Proca field coupled to dissipative permeability tensors, manifestly covariantly in a frequency-space representation. In contrast to non-dissipative real permeabilities, the complex dispersion relations unambiguously determine the retarded/advanced Green function without epsilon regularization. We derive the energy conservation law for the dissipating radiation field, and identify the spectral densities of field energy and Poynting vector by time averaging. In this way, we can also quantify the energy absorption induced by the complex permeabilities and the complex tachyonic mass-square in the field equations. The attenuation lengths defining the damping factor in the classical tachyonic Cherenkov densities differ for transversal and longitudinal radiation. We average these densities over a relativistic electron gas and put them to test by performing a spectral fit to the supernova remnant RX J1713.7 − 3946, whose γ-ray flux has been measured by the Fermi satellite [10] and the ground-based atmospheric imaging array HESS [11,12]. The spectral fit covers five frequency decades, the GeV range up to 100 TeV.

In Section 2, we introduce the tachyonic Maxwell–Proca Lagrangian in the absorptive spacetime aether described by complex frequency-dependent permeability tensors. We explain the meaning of the complex Lagrangian and the manifestly covariant action functional, and derive the field equations and the constitutive relations, manifestly covariantly and also in 3D. In Section 3, we derive the continuity equation for the field energy in the aether defined by homogeneous and isotropic permeabilities, and extract the spectral energy flux by time averaging. In Section 4, we separate the transversal and longitudinal components of the tachyonic radiation field to obtain explicit formulas for the dissipating transversal/longitudinal energy and flux densities.

In Section 5, we derive the dispersion relations from the wave equations for the transversal and longitudinal vector potentials, as well as asymptotic formulas for the complex transversal/longitudinal wavenumbers by ascending series expansion in the imaginary parts of the permeabilities. In Section 6, we discuss spherical

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http://dx.doi.org/10.1016/j.physleta.2014.08.010
0375-9601/

wave propagation in the aether, employing transversal/longitudinal Green functions in space-frequency representation. In contrast to a non-absorptive spacetime, the Green function, retarded or advanced, is already determined by the imaginary part of the dispersion relation, without the use of epsilon regularization of pole singularities prescribing residual integration paths. We use the retarded Green function in dipole approximation to obtain the attenuated radiation fields at large distance from the localized source.

In Section 7, we calculate the classical tachyonic Cherenkov densities of an inertial subluminal charge in the dissipative aether. There are two different ways to derive the Cherenkov effect. In Fermi's approach, one considers the radiating charge moving along an infinite straight line, solves the field equations in cylindrical coordinates, and calculates the flux streaming orthogonally through a cylinder around the particle trajectory [3,4]. A preferable method due to Tamm is to consider the trajectory within a finite time interval, so that the current distribution is compact. The time-averaged asymptotic flux through a large sphere is then calculated in dipole approximation, and the averaging period is finally extended to infinity to remove the bremsstrahlung contribution occurring at the end points of the truncated trajectory [13,14]. This bremsstrahlung vanishes with increasing averaging period and is not to be confused with photonic bremsstrahlung which can arise in the weakly coupled plasma of the remnant [15–17]. The asymptotic spherical symmetry of Tamm's method is better adapted to the radiation problem studied here than an infinite cylindrical geometry. In Section 8, we average the transversal/longitudinal radiation densities over an ultra-relativistic thermal electron gas and perform a tachyonic Cherenkov fit to the γ-ray emission of SNR RX J1713.7 − 3946. In Section 9, we present our conclusions.

2. Maxwell–Proca Lagrangian in an absorptive spacetime

Throughout this article, we use a frequency-space representation of the real Proca field $\hat{A}_\mu(\mathbf{x},\omega) = \int_{-\infty}^{\infty} A_\mu(\mathbf{x},t)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}t$, suitable for dispersive and dissipative permeabilities [18–21]. Time Fourier transforms are denoted by a hat, and the reality condition is $\hat{A}^*_\mu(\mathbf{x},\omega) = \hat{A}_\mu(\mathbf{x},-\omega)$. We start with the formally manifestly covariant Maxwell–Proca Lagrangian

$$\hat{L} = -\frac{1}{4}\hat{F}^*_{\mu\nu} g_F^{\mu\alpha} g_F^{\nu\beta} \hat{F}_{\alpha\beta} + \frac{1}{2} m_\mathrm{t}^2 \hat{A}^*_\mu g_A^{\mu\nu} \hat{A}_\nu + \frac{1}{2}\left(\hat{A}^*_\mu g_J^{\mu\nu} \hat{j}_\nu + \hat{A}_\mu g_J^{\mu\nu} \hat{j}^*_\nu\right), \tag{2.1}$$

where $\hat{F}_{\mu\nu}(\mathbf{x},\omega)$ is the Fourier transform of the field tensor $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$. Time differentiation in Fourier space means to multiply with a factor $-\mathrm{i}\omega$, e.g. $\hat{A}_{\mu,0} = -\mathrm{i}\omega\hat{A}_\mu$ and $\hat{A}^*_{\mu,0} = \mathrm{i}\omega\hat{A}^*_\mu$ for conjugated fields. Greek indices are raised and lowered with the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. The permeability tensors $g^{\mu\nu}_{A,F,J}(\omega)$ are homogeneous and isotropic, satisfying the reality condition $g^{*\mu\nu}_{A,F,J}(\omega) = g^{\mu\nu}_{A,F,J}(-\omega)$ like the complex permeabilities $(\varepsilon_0(\omega), \mu_0(\omega))$, $(\varepsilon(\omega), \mu(\omega))$ and $\Omega(\omega)$ defining them,

$$g_A^{00} = -\varepsilon_0, \qquad g_A^{ij} = \frac{\delta^{ij}}{\mu_0},$$
$$g_F^{00} = -\mu^{1/2}\varepsilon, \qquad g_F^{ij} = \frac{\delta^{ij}}{\mu^{1/2}}, \tag{2.2}$$

and $g^{0i}_{A,F} = 0$. The tensor $g_J^{\mu\nu}(\omega) = \eta^{\mu\nu}/\Omega(\omega)$ is conformal to the Minkowski metric and amounts to a frequency-dependent coupling constant. All permeabilities have a positive real part, and principal values are assumed for roots. We use the Heaviside–Lorentz system, so that $\varepsilon = \varepsilon_0 = 1$ and $\mu = \mu_0 = 1$ in vacuum. The complex frequency-dependent tachyon mass $m_\mathrm{t}(\omega)$ in the Lagrangian satisfies $m_\mathrm{t}(-\omega) = m^*_\mathrm{t}(\omega)$, with positive real part. The mass-square can be scaled into $g_A^{\mu\nu}(\omega)$, cf. Section 3. The external current is conserved, $\hat{j}^\nu_{,\nu} = 0$ (that is $\hat{j}^m_{,m} - \mathrm{i}\omega\hat{j}^0 = 0$) and satisfies the reality condition. Lagrangian (2.1) is real only if the permeabilities and the tachyon mass are real, in the absence of absorption.

We define the inductive fields $\hat{H}^{\alpha\beta} = g_F^{\alpha\mu} g_F^{\beta\nu}\hat{F}_{\mu\nu}$ and $\hat{C}^\mu = g_A^{\mu\nu}\hat{A}_\nu$, as well as the dressed current $\hat{j}^\mu_\Omega = g_J^{\mu\nu}\hat{j}_\nu$, which are the manifestly covariant constitutive relations in the absolute spacetime. Euler variation of the Lagrangian with respect to $\hat{A}^*_\mu$ gives the field equations $\hat{H}^{\mu\nu}_{\ \ ,\nu} - m_\mathrm{t}^2\hat{C}^\mu = \hat{j}^\mu_\Omega$. Differentiation followed by contraction leads to the Lorentz condition $\hat{C}^\mu_{,\mu} = 0$, as the current is conserved. Variation of the Lagrangian with respect to $\hat{A}_\mu$ results in a different set of field equations, where the permeability tensors and the mass-square are replaced by the conjugated quantities, $g^{\mu\nu}_{A,F,J}(\omega) \to g^{*\mu\nu}_{A,F,J}(\omega)$, $m_\mathrm{t}(\omega) \to m^*_\mathrm{t}(\omega)$. The imaginary parts of the permeability tensors and the tachyon mass determine whether the Green function is retarded or advanced, cf. Sections 5.2 and 6.1. (In contrast to real permeabilities, there are no poles on the real axis to be circumvented by epsilon regularization, that is, by an infinitesimal $\pm\,\mathrm{sign}(\omega)\mathrm{i}\varepsilon$ or $\pm\mathrm{i}\varepsilon$ insertion in the denominator of the Green function in momentum space, cf. Section 6.) For any given frequency ω, either $(g^{\mu\nu}_{A,F,J}, m_\mathrm{t})$ or the conjugated quantities $(g^{*\mu\nu}_{A,F,J}, m^*_\mathrm{t})$ define retarded wave propagation, and we use the corresponding wave equation at this frequency. For the sake of definiteness, we will consider a frequency interval in which $(g^{\mu\nu}_{A,F,J}, m_\mathrm{t})$ gives retarded propagation.

If the permeability tensors and the tachyon mass are real and constant (frequency-independent), we can use a spacetime representation of the Lagrangian,

$$L = -\frac{1}{4}F_{\mu\nu}H^{\mu\nu} + \frac{1}{2}m_\mathrm{t}^2 A_\mu C^\mu + A_\mu j^\mu_\Omega, \tag{2.3}$$

resulting in the action $S = \int L\mathrm{d}\mathbf{x}\mathrm{d}t = (2\pi)^{-1}\int \hat{L}\mathrm{d}\mathbf{x}\mathrm{d}\omega$, with $\hat{L}$ in (2.1). In the case of complex frequency-dependent permeabilities, we employ the second identity to define the action. S is real since $\hat{L}(\mathbf{x},-\omega) = \hat{L}^*(\mathbf{x},\omega)$ and does not change if we replace Lagrangian (2.1) by its complex conjugate, which means to replace $(g^{\mu\nu}_{A,F,J}, m_\mathrm{t})$ by $(g^{*\mu\nu}_{A,F,J}, m^*_\mathrm{t})$.

The 3D field strengths are $\hat{E}_k = \hat{F}_{k0} = \mathrm{i}\omega A_k + A_{0,k}$ and $\hat{B}^k = \varepsilon^{kij}\hat{F}_{ij}/2 = \varepsilon^{kij}\hat{A}_{j,i}$, and inversely $\hat{F}_{ij} = \varepsilon_{ijk}\hat{B}^k$, where ε^{kij} is the Levi-Civita 3-tensor. The 3D inductions are defined by the constitutive relations $\hat{D}^l = \hat{H}^{0l} = \varepsilon\hat{E}^l$ and $\hat{H}_i = \varepsilon_{ikl}\hat{H}^{kl}/2 = \hat{B}_i/\mu$, as well as $\hat{C}_m = \hat{A}_m/\mu_0$ and $\hat{C}_0 = \varepsilon_0\hat{A}_0$ for the potential, cf. after (2.2). The charge densities $\hat{\rho} = \hat{j}^0$ and $\hat{\rho}_\Omega = \hat{j}^0_\Omega$ of external and dressed current are related by $\hat{\rho}_\Omega = \hat{\rho}/\Omega$, and the respective 3-currents by $\hat{j}^k_\Omega = \hat{j}^k/\Omega$. The 3D inhomogeneous field equations read

$$\varepsilon^{klj}\hat{H}_{j,l} + \mathrm{i}\omega\hat{D}^k - m_\mathrm{t}^2\hat{C}^k = \hat{j}^k_\Omega, \qquad \hat{D}^l_{,l} - m_\mathrm{t}^2\hat{C}^0 = \hat{j}^0_\Omega. \tag{2.4}$$

The homogeneous Maxwell equations follow from the above potential representation of the field strengths, and we also mention conservation of the dressed current and the Lorentz condition,

$$\varepsilon^{ikn}\hat{E}_{n,k} - \mathrm{i}\omega\hat{B}^i = 0, \qquad \hat{B}^i_{,i} = 0,$$
$$\mathrm{i}\omega\hat{j}^0_\Omega - \hat{j}^k_{\Omega,k} = 0, \qquad \mathrm{i}\omega\hat{C}^0 - \hat{C}^l_{,l} = 0. \tag{2.5}$$

The rising and lowering of the zero index is accompanied by a sign change, as we use the sign convention $\mathrm{diag}(-1,1,1,1)$ for the Minkowski metric.

3. Tachyonic Poynting vector and dissipative energy density

To derive the energy conservation law in an absorptive spacetime [22–25], we employ the inhomogeneous field equations (2.4),