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A phenomenological memristor model for short-term/long-term memory

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ABSTRACT

Memristor is considered to be a natural electrical synapse because of its distinct memory property and nanoscale. In recent years, more and more similar behaviors are observed between memristors and biological synapse, e.g., short-term memory (STM) and long-term memory (LTM). The traditional mathematical models are unable to capture the new emerging behaviors. In this article, an updated phenomenological model based on the model of the Hewlett–Packard (HP) Labs has been proposed to capture such new behaviors. The new dynamical memristor model with an improved ion diffusion term can emulate the synapse behavior with forgetting effect, and exhibit the transformation between the STM and the LTM. Further, this model can be used in building new type of neural networks with forgetting ability like biological systems, and it is verified by our experiment with Hopfield neural network.

1. Introduction

Memristor was theoretically envisioned by Prof. Chua back in 1971 [1], as the fourth basic circuit element besides resistor, capacitor and inductor [1]. The device didn't attract attention until the Hewlett–Packard (HP) realized the physical device in a cross-bar array structure in 2008 [2]. Its distinct properties, such as nanoscale, low energy dissipation, and the memory ability make it a prospective application in the design and optimization of circuits [3–9]. In the brain-like neuromorphic circuits, memristor replaces the traditional circuit structures, which consist of transistors and capacitors, to serve as synapse to transmit information between neurons. Analogous to the plasticity of biological synapse, memristor can change its memristance by the historic current through itself [2,10,11]. Besides, other similar properties are also observed in memristors, e.g., the spike-timing-dependent plasticity (STDP) and the 'learning-experience' behavior in the α-IGZO memristor [12], the spike-number-dependent plasticity in the Ag_2S memristor [13]. Compared with those properties, a more general and interesting synaptic function is the short-term memory (STM) and the long-term memory (LTM) plasticity simultaneously observed in the WO_x memristor [14], the Ag_2S [15] memristor and the α-IGZO memristor [12].

The concept of the STM and the LTM originates from the psychological model of human memory proposed by Atkinson and Shiffrin as shown in Fig. 1 [15]. The STM generally lasts a short period while the LTM lasts a much longer time. Despite the presence of natural forgetting, the STM can result in the LTM by repeated rehearsal, i.e., the consolidation process. Meanwhile, the LTM can also transfer to the STM by the retrieval process. Along with the process of the STM and the LTM, a natural forgetting is lasting even without any external stimulus. The forgetting ability is necessary for the system to release space for the truly useful information, after all, the capacity of human memory is limited [14]. Because the multi-store model is the basic part in learning and decision-making in biological systems, the forgetting ability and the STM/LTM plasticity of memristor become significant for electrical synapse in mimicking the biological synapse.

Memristors with different material always have different properties, and even the same memristor with different fabricating temperature might have different properties [13]. As a result, there are a lot of mathematical models to emulate different behaviors of different memristor [16–21]. Some of the previous models are developed from the physical mechanism, such as the HP memristor model based on ion drift [17]. Some are built by mathematical skills, such as the piecewise linear model [18]. The former ones always focus only on the behavior of the corresponding physical

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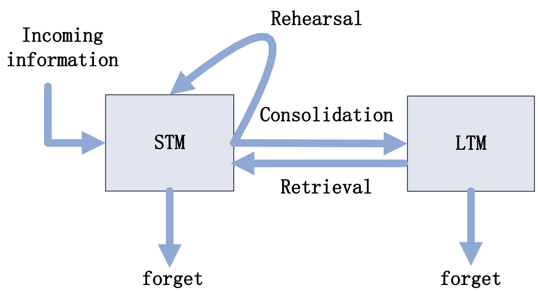

Fig. 1. The multi-store model about the short-term memory and the long-term memory.

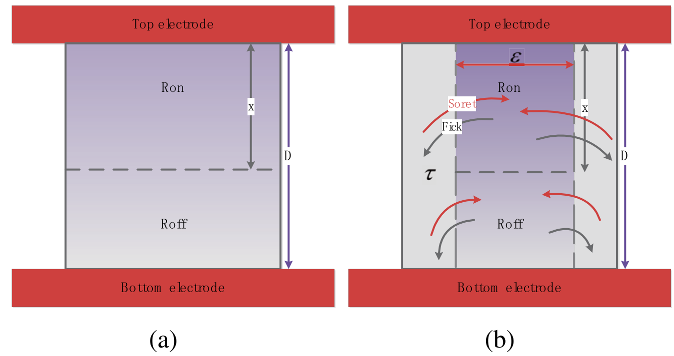

Fig. 2. (a) The mechanism of the old HP model. (b) The mechanism of the new model.

memristors, for example, the HP memristor model built according to the ion migration theory could not explain the behavior of WO_x memristor supported by the theory of Schottky barrier and ohmic-like contact in [22]. The latter ones are either weak in data matching or powerful in data matching but with a lot of fitting parameters, such as the threshold model in [19] and the mathematical model in [20]. Nevertheless, all these models can't describe the new STM/LTM property. Therefore, in this article, we focus on the STM/LTM property to develop a new mathematical model. The contribution of this article is:

1. We make a supplement on the previous ion drift theory by adding the consideration of the Fick ion diffusion (the concentration gradient induced diffusion) and the Soret diffusion (the thermal diffusion) [23]. The new ion drift theory is more reasonable and more general.
2. We develop a new model based on the previous HP memristor model to capture the STM and the LTM property observed in physical memristors.
3. The new model describes the forgetting effect and the spike-rate-dependent property of memristor, besides the STM and the LTM plasticity.
4. The new model can solve the boundary effect of all window functions discussed in [17].
5. We build a new Hopfield neural network with this new memristor model, and the neural network can mimic the forgetting ability of human being.

The structure of this article is as follows. In Section 2, we propose a new model with a HP ion migration term and an improved ion diffusion term. Section 3 is devoted to emulate the synaptic functions including the transformation between the STM and the LTM by the new memristor model. In Section 4, we build a Hopfield neural network with the new memristor model to mimic the forgetting effect in the human memory. Section 5 is a conclusion of this article, and in this section, we discuss some pending topics about this article.

2. The new HP model with improved ion diffusion term

The HP model [17] is a classical model to describe the ion migration behavior in memristor with a few parameters. It divides the memristor into two parts, the doped and the undoped area, by the drift boundary of the oxygen vacancy, and takes an inner state variable x to represent the length of doped area as shown in Fig. 2(a). In general, the ion drift caused by the electric field is the dominant activity in the memristor, but in some cases, some other activities such as the Fick diffusion and the Soret diffusion can become the dominant factor influencing the oxygen vacancy motion. Ting Chang et al. proposed an diffusion term $-\frac{x}{\tau}$ (x is the inner state variable, τ is the decaying rate, the larger it is, the more time it needs to decay) to mimic the behavior of ion diffusion in the Pd/WO_x/W (top electrode/memristor/bottom electrode) memristor in [22], but that ion diffusion term has a limitation on describing the transformation between the STM and the LTM. In our previous work [24], we have improved the ion diffusion term to be $-\frac{x-\varepsilon}{\tau}$ (ε is the retention) for a better matching with the physical Pd/WO_3/W memristor. In the improved ion diffusion term, x is the indication of the Fick diffusion, while the ε is the indication of the Soret diffusion. As we know, the Fick diffusion will always decrease the conductivity because it will make oxygen vacancy run away from the conductive channel, which has a higher concentration than other parts, while the Soret diffusion will always increase the conductivity because it will attract more oxygen vacancy diffusing into the conductive channel, which always has a higher temperature due to the Joule heating. If we take the ion diffusion into account, the HP model will be described as Fig. 2(b). The conductive area is a variable-width-length channel in the film. The width and the length are represented by ε and x respectively, both of them are affected by the electric field in a similar way. Both the Fick diffusion and the Soret diffusion are positively affected by the electric field, but their directions are independent, as the black and the red arrows marked in Fig. 2(b). τ is the final decaying rate deterred by the thermal diffusivity and the Fick diffusion coefficient, which are dependent on the material. Here we assume that they are variables and the thermal diffusivity is concerned with the electric field (or temperature) while the Fick diffusion coefficient is concerned with both the electric field and the concentration [25,26]. The old HP memristor model is weak in data matching, but it is abstract and convenient for mathematical analysis. So we take it as the ion drift term, and we adopt our previous ion diffusion term to describe the ion diffusion. Thus, the new model for oxide memristors is designed as:

$$i = \frac{v}{R_m}, \tag{1}$$

$$R_m = R_{on}x + R_{off}(1-x), \tag{2}$$

$$\frac{dx}{dt} = \left(l - \frac{x-\varepsilon}{\tau}\right) f(x), \quad l = \frac{\mu_v R_{on}}{D^2} i(t), \tag{3}$$

$$\frac{d\varepsilon}{dt} = \sigma l f(x), \tag{4}$$

$$\frac{d\tau}{dt} = \theta l(a-x), \tag{5}$$

where v is the voltage of the memristor, i is the current passing through it, l is the ion migration term derived from the old HP memristor model [17]. In our model, R_m is the memristance, R_{on} is the minimum memristance which means all the film is doped, R_{off} is the maximum memristance which means all the film is undoped, D is the thickness of the memristor film, μ_v is the drift rate of oxygen vacancy. Besides the above general parameters of