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Blood flow of Jeffrey fluid in a catherized tapered artery with the suspension of nanoparticles

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ABSTRACT

Current letter deals with the mathematical models of Jeffrey fluid via nanoparticles in the tapered stenosed atherosclerotic arteries. The convection effects of heat transfer with catheter are also taken into account. The nonlinear coupled equations of nanofluid model are simplified under mild stenosis. The solutions for concentration and temperature are found by using homotopy perturbation method, whereas for velocity profile the exact solution is calculated. Moreover, the expressions for flow impedance and pressure rise are computed and discussed through graphs for different physical quantities of interest. The streamlines have also been presented to discuss the trapping bolus discipline.

1. Introduction

It is believed that abnormal growth of arterial thickness is an early process in the formation of atherosclerosis and one of the most widespread diseases in humans. Atherosclerosis is a major underlying cause of angina and myocardial infarction [1]. A partial or total circulatory occlusion in a coronary artery reduces the blood supply to the heart and the vascular wall experiences considerable stiffness by the buildup of plaque with lipid core and a fibromascular cap, which increases the heart attack probabilities. The analysis of blood flow through tapered tubes is very important in understanding the behavior of the flow as the taper of the tube is an important factor in the pressure development. The accumulation of substances in arteries is known as stenosis. Stenosis narrows the artery because of blood has to pass with relatively high pressure. To diagnose and handle such diseases, catheter is used as a standard tool. It changes the flow pattern and hemodynamic conditions that exist in the artery before catheterization. Catheters can be inserted into a body cavity, duct, or vessel. They perform wide variety of tasks depending on the type of catheter. The arterial walls may be elastic, movable or permeable. Most of the biological organs contain a permeable layer attached to the wall, for instance, blood vessel consists of tissue layer. Recent developments in the blood flow characteristics are indispensable in order to have a fuller understanding of the stenosis and permeable layer. The important contributions to the topic are referenced in the literature [2–4].

Moreover, in most of the studies, flowing blood is assumed to be Newtonian. The assumption of Newtonian behavior of blood is acceptable for high shear rate flow, i.e. the case of flow through larger arteries. It is not, however, valid when the shear rate is low as is the flow in smaller arteries and in the downstream of the stenosis. It has been pointed out that in some diseased conditions, blood exhibits remarkable non-Newtonian properties since it is seen through experiments that most of the biological fluids exhibit rheology of non-Newtonian characteristics. Jeffrey fluid is specified as a non-Newtonian fluid having shear thinning property for which viscosity of fluid reduces with increasing rate of shear stress. Mekheimer and El Kot [5] have studied the mathematical modeling of unsteady flow of fluid through anisotropically tapered elastic arteries with time variant overlapping stenosis. They analytically solved their mathematical model for mild stenosis case. Riahi et al. [6] have examined the problem of blood flow in an artery and in the presence of an overlapping stenosis. Srivasta et al. explored the blood flow through a composite stenosis in catheterized arteries on the assumption that blood behaves as Newtonian fluid

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[7]. Some important studies dealing with different models of Newtonian and non-Newtonian fluid are given in Refs. [8–14].

Furthermore, recent advances in nanotechnology have led to development of a new innovative class of heat transfer called nanofluids which is created by dispersing nanoparticles in traditional heat transfer fluids. Nanofluid has been found to possess enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids like oil or water. Non-Newtonian nanofluids are widely encountered in many biological applications such as nano-drug delivery, cancer therapeutics, cryopreservation oil, water, and ethylene glycol mixture. Some relevant studies on the topic can be seen from the list of references [15–17].

Motivated by these facts, a successful attempt is made to discuss the nanoparticles effects in tapered artery with stenosis in order to analyze the fully developed flow of an incompressible, Jeffrey fluid along with the catheter. Convection effect of heat transfer is also taken into account. It is very easy to solve a linear problem, but finding a solution of nonlinear problem is still a very challenging task. Despite the availability of high performance supercomputers, getting an analytic and semianalytical solutions of a nonlinear problems is often more difficult compared to getting a numerical solution. To drive the solutions of nonlinear equations we have used one of the most modern methods, Homotopy Perturbation Method (HPM), which has already been successfully applied for strongly nonlinear problems [18–20]. The work undertaken is a blend of analytical and semianalytical studies. The physical features of the major parameters are discussed through graphs and the trapping phenomenon has also been presented at the end through streamlines.

2. Formulation of the problem

Consider an incompressible Jeffrey fluid with nano-particles flowing through catheterized, tapered artery of finite length L with overlapping stenosis. Let (r, θ, z) be the coordinates of a material point in the cylindrical polar coordinate system, where z-axis is taken along the axis of artery, while r, θ are along the radial and circumferential direction respectively. Moreover, $r = 0$ is taken as the axis of symmetry of the tube. Heat and mass transfer phenomenon are taken into account by giving temperature T_1 and concentration C_1 to the wall of the tube. At the centre of the tube, we consider symmetric conditions for velocity, temperature and concentration. The geometry of the arterial wall of the overlapping stenosis for different tapering angles is written mathematically

$$R(z) = d(z)\left[1 - \psi\left(L_0^{n-1}(z - d_0) - (z - d_0)^n\right)\right],$$
$$d_0 < z \le d_0 + L_0, \tag{1}$$
$$R(z) = d(z), \quad \text{otherwise} \tag{2}$$

with

$$\psi = \frac{\delta n^{\frac{n}{n-1}}}{R_0 L_0^n (n-1)}, \tag{3}$$
$$d(z) = R_0 + \xi z, \tag{4}$$

in which δ denotes the maximum height of the stenosis located at

$$z = d_0 + \frac{L_0}{n^{\frac{n-1}{n}}}. \tag{5}$$

Here R_0 is the radius of the non-tapered artery in the non-stenotic region, $d(z)$ is the radius of the tapered arterial segment in the stenotic region, ξ is the tapering parameter, L_0 is the length of the stenosis, n ($\ge$2) is a parameter determining the shape of the

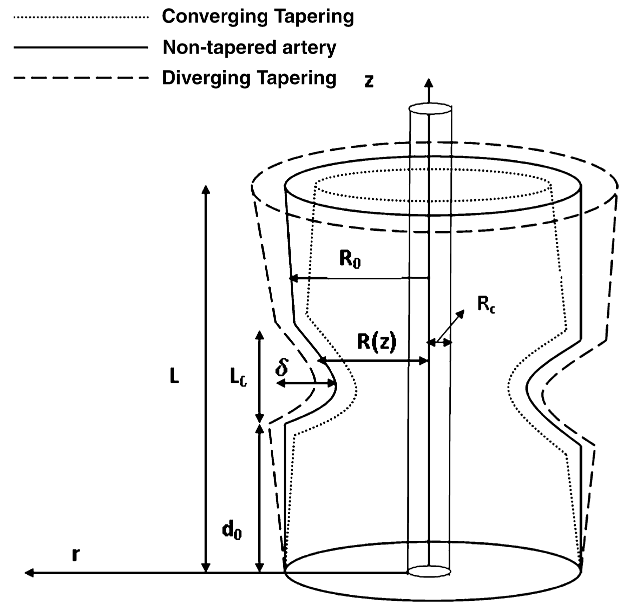

Fig. 1. Geometry of the stenosed tapered artery for different tapering angle.

constriction profile, referred to as the shape parameter for which symmetric stenosis is found for $n = 2$ and d_0 indicates location of stenosis, as shown in Fig. 1.

The equations for unsteady flow of an incompressible nanofluid in the presence of body force are given by

$$\frac{1}{r}\frac{\partial(rv)}{\partial r} + \frac{\partial u}{\partial z} = 0, \tag{6}$$

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) - \frac{1}{r}S_{\theta\theta}, \tag{7}$$

$$\rho\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) + \rho g\alpha_1(T - T_1) + \rho g\alpha_1(C - C_1), \tag{8}$$

$$\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = \alpha_1\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial r}\frac{\partial T}{\partial r} + \frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right) + \frac{D_T}{T_0}\left(\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right)\right], \tag{9}$$

$$\left(v\frac{\partial C}{\partial r} + u\frac{\partial C}{\partial z}\right) = D_B\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right) + \frac{D_T}{T_0}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right), \tag{10}$$

where $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle and heat capacity of the fluid, c is the volumetric volume expansion coefficient, $\mathbf{V}$ is the velocity vector, $\mathbf{f}$ is the body force, d/dt represents the material time derivative, p is the pressure, C is the nanoparticle phenomena. The ambient values of T and C as r tent to R are denoted by T_1 and C_1, D_B is the Brownian diffusion coefficient and D_T is the thermospheric diffusion coefficient.

$$S_{rr} = \frac{2\mu}{1+\lambda_1}\left(1 + \lambda_2\left(v\frac{\partial}{\partial r} + u\frac{\partial}{\partial z}\right)\right)\frac{\partial v}{\partial r}, \tag{11}$$

$$S_{rz} = \frac{\mu}{1+\lambda_1}\left(1 + \lambda_2\left(v\frac{\partial}{\partial r} + u\frac{\partial}{\partial z}\right)\right)\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right), \tag{12}$$