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The estimation of a unique solution for steady-state diffuse optical tomography by applying mechanical pressure

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ABSTRACT

The accuracy of diffuse optical tomography (DOT) highly depends on two important factors: first, the knowledge of the tissue optical heterogeneities for accurate modeling of light propagation, and second, the uniqueness of reconstructed values of optical properties. Previous studies illustrated that the inverse problem associated with steady-state DOT does not have unique solutions. In this study, we propose a simple method that can be applied to improve this challenging problem of steady-state DOT. In this method, we study the propagation of photons through compressed breast phantoms. The applied mechanical pressure can change the values of optical properties and this pressure dependence of optical properties as a set of constraint equations can be used to improve the inverse problem. The applied pressure can help us to restrict the distribution of possible values of depth and radius of defect inside breast phantom reconstructed by inverse problem.

Diffuse optical tomography (DOT) is an optical imaging modality that is applied for monitoring of molecular and cellular activities in cancer and infant brain imaging studies [1–3]. There are currently three approaches for achieving DOT: steady-state, time domain, and frequency domain. In steady-state DOT, the intensity of optical source is steady-state and the variation of intensity is considered to inverse problem. The steady-state DOT is a low-cost imaging method that has become gradually accepted for monitoring neural activities [4–6]. However, the accuracy and precision of steady-state DOT is a challenging problem. As discussed in Ref. [7], DOT based only on intensity is a non-unique inverse problem, meaning that the recovery of absorption and diffusion coefficient is not feasible [8]. This important challenge of steady-state DOT limits the applications of this low-cost imaging system. In 2003, Corlu and his colleagues presented special conditions for the unique and simultaneous recovery of chromophore concentrations and scattering coefficients in multispectral steady-state DOT. Their approach makes three assumptions. First, the wavelength-dependent reduced-scattering coefficient $\sigma'(\lambda)$ is assumed to take on a simplified Mie-scattering form, $\sigma'(\lambda) = \alpha\lambda^{-b}$, where α and b are related to the size, the index of refraction, and the concentration of scatterers in the tissue as well as the index of refraction of the surrounding medium [8]. Second, a unique solution can be achieved by using appropriate spectral illumination line. Third, the number of wavelengths is greater than the number of chromophores, namely the number of equations is greater than the number of unknowns. The first and second assumptions of this method restrict application of this method because the presented wavelength dependence of reduced scattering coefficient is based on Mie theory which can be roughly applied for normal breast tissue and this theory cannot explain the scattering properties of different biological tissues. Moreover, choosing appropriate wavelengths for different biological tissue may be challengeable.

Harrach in 2009 demonstrated that it suffices to restrict ourselves to piecewise constant diffusion and piecewise analytic absorption coefficients to regain uniqueness. Under this condition both parameters can simultaneously be determined from complete measurement data on an arbitrarily small part of the boundary [9]. Jiang in 2011 stated that in DOT, theoretically infinite absorption and reduced scattering coefficient solution exist, but practically all solutions that significantly differ from the exact solution have been excluded with *a priori* knowledge on optical properties of tissue [10]. The results presented by Corlu and Jiang show that the unique solution in steady-state DOT can be estimated by *a priori* knowledge or using multispectral DOT.

In this study, based on theorems verified by Harrach and results presented in Ref. [10], we propose a simple way for the accurate recovery of values of depth and radius of defect in breast phantom. As presented in Refs. [1,8], one can increase the number of equations to restrict the number of solutions. In this Letter, we present a simple way to generate a number of constraint equations which can be used to recovery unique solution of optical heterogeneity

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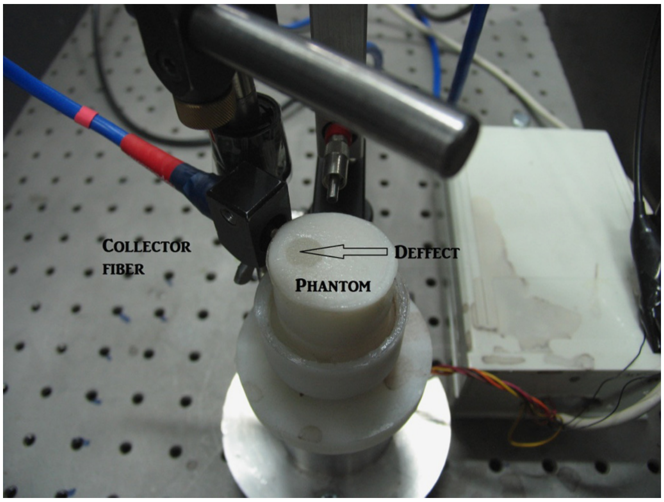

Fig. 1. The picture of inhomogeneous phantom in low-cost steady-state DOT setup.

localization in biological breast phantom. This method has been evaluated by simulation and experimental studies. We estimate a single value for size and depth of defect in breast phantom.

The presented method is based on simulation of diffused photons in compressed phantom, namely the variation of optical properties induced by mechanical pressure is used to improve the procedure of inverse problem. The applied pressure can generate a number of constraint equations which can help us to restrict solutions. In previous study, we illustrated that applied mechanical pressure can change optical properties of breast phantom [11] and the pressure dependence of optical properties can be considered as a modification in procedure of inverse problem. In this Letter, we show that the consideration of effects of applied pressure can restrict the distribution of possible reconstructed data.

In the steady-state DOT and based on results presented by Harrach, the diffusion of photons inside biological tissue is modeled by diffusion equation:

$$-D\nabla^2\varphi(\vec{r}) + a\varphi(\vec{r}) = S(\vec{r}) \tag{1}$$

Here, $\vec{r}$ denotes position, $\varphi(\vec{r})$ is the fluence rate defined as the energy flow per unit area per unit time. $S(\vec{r})$ is the isotropic source term at position $\vec{r}$. D refers to diffusion coefficient:

$$D = \frac{1}{3(a+\sigma')} \tag{2}$$

where a and σ' are absorption and reduced scattering coefficient, respectively. As shown in Fig. 1, the phantom is composed by two subdomains, where D is constant in each subdomain. By applying mechanical pressure ΔP, φ is changed to $\varphi' = \varphi + \frac{\partial\varphi}{\partial P}\Delta P$. The absorption and diffusion coefficients are changed to $a' = a + \frac{\partial a}{\partial P}\Delta P$ and $D' = D + \frac{\partial D}{\partial P}\Delta P$. By differentiating Eq. (1) with respect to pressure and with some mathematics, we obtain:

$$-D'\nabla^2\varphi'(\vec{r}) + a'\varphi'(\vec{r}) = S(\vec{r}) \tag{3}$$

By applying different mechanical pressures, Eq. (3) as constraint equation can be simultaneously solved with Eq. (1). For each applied pressure P_i, the values (D'_i, a'_i) depend on four unknown values as follows:

$$\begin{cases} D'_i = D + \dfrac{\partial D}{\partial P}\Delta P_i \\ a'_i = a + \dfrac{\partial a}{\partial P}\Delta P_i \end{cases} \quad i = 1, \ldots, n \tag{4}$$

where n is the number of applied pressures. Therefore, unknown optical parameter (D, a) with infinite possible solution must be satisfied by the constraint conditions stated in Eq. (4). Based on Harrach postulated presented in Ref. [9], these optical properties can be recovered for $P = 0, \ldots, P_n$. The unknown values $\frac{\partial D}{\partial P}$ and $\frac{\partial a}{\partial P}$ can be represented by matrix $X = \{[\frac{\partial D}{\partial P}]_{2\times1}, [\frac{\partial a}{\partial P}]_{2\times1}\}^T$ as $AX = B$, that $A = \{\Delta P_i\}_{2n\times1}$ and $B = \{[D'_i - D]_{m\times n}, [a'_i - a]_{m\times n}\}$, where m is the number of subdomain of phantom. With $(n > m)$ this linear equation has a solution in least square sense, $X_0 = (A^TA)^{-1}A^TB$, and the residual norm $R = \|B - AX_0\|$ can be interpreted as the parameter of distinguishability. X_0 is closer to fulfill the conditions for uniqueness as R approaches to zero. By the considering Harrach's assumption presented in Ref. [9], the nonlinear Tikhonov regularization, and the tiny value R ensure the uniqueness of solution in steady-state DOT.

To evaluate this method, first the diffusion equation with Robin boundary condition

$$\varphi(\vec{r}) - 2AD\vec{n}.\nabla\varphi(\vec{r}) = 0 \tag{5}$$

should be solved, where $\vec{n}$ is the normal vector for the boundary, and $A = (1 + r_d)/(1 - r_d)$, where r_d can be approximated by $r_d \approx -1.4399n_{rel}^{-2} + 0.7099n_{rel}^{-1} + 0.668 + 0.0636n_{rel}$, where n_{rel} is the ratio of refractive indices. For a value of $n_{rel} = 1.33$ we get $A = 2.82$ [12,13]. The diffusion equation with Robin boundary condition can be solved by fundamental Green's function. In this method, the differential equation is converted to an integral equation called the boundary integral equation (BIM) and then this integral equation is solved by boundary element method (BEM). We have explained this method in Ref. [11]. As mentioned in Ref. [14], image reconstruction in DOT involves both the forward and inverse problems. The forward problem usually used the diffusion equation to predict the distributions of reemitted light on the basis of presumed parameters for both the light sources and the object. In this study, the process of image reconstruction is based on the BIM forward method for solving the diffusion equation for imaging domains. The reconstruction process solves an inverse problem to determine the optical properties of tissue or the location of the in-homogeneous defect. The image reconstruction is achieved through an iterative procedure where an objective function consisting of the difference between the measured and the modeled data is minimized. In our case, the least-squares functional to be minimized is [10]:

$$F^2 = \sum_{j=1}^{s}\left(\varphi_j'^{\,measured} - \varphi_j'^{\,cal}\right)^2 \tag{6}$$

where $\varphi_j'^{\,measured}$ and $\varphi_j'^{\,cal}$ are the measured and calculated fluences at the measurement point j, respectively. s is the total number of measurements. The procedure of inverse model is based on details presented in Ref. [11]. In this study, to compare the measured value obtained by steady-state DOT with expected values, we use the correlation coefficient R_c that can be calculated as follows:

$$R_c = \left(\frac{p\sum xy - (\sum x)(\sum y)}{\sqrt{p(\sum x^2) - (\sum x)^2}\sqrt{p(\sum y^2) - (\sum x)^2}}\right)^2 \tag{7}$$

where x and y are the expected and measured values, respectively, and p is the number of data. R_c is a measure that determines the degree of coherency and correlation between expected value and measured values obtained by steady-state DOT. A correlation coefficient close to unit indicates a good association between variables. In other words, the value of correlation coefficient close to unit results in a linear correlation between variables x and y.