Physics Letters A 373 (2009) 2911–2916

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

The pushing gate in a planar Coulomb crystal using a flat-top laser beam

M. Kitaoka [a,*], I.M. Buluta [b,1], S. Hasegawa [a,b]

[a] Department of Systems Innovation, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
[b] Department of Quantum Engineering and Systems Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

ARTICLE INFO

Article history:
Received 25 February 2009
Received in revised form 11 May 2009
Accepted 9 June 2009
Available online 12 June 2009
Communicated by P.R. Holland

Keywords:
Ion trap
Quantum computation
Quantum simulation
Pushing gate
Calcium ion

ABSTRACT

We propose a pushing gate for entangling two ions in a planar Coulomb crystal in the view of realizing large-scale quantum simulations. A tightly focused laser is irradiated from the direction perpendicular to the crystal plane and its spatial intensity profile generates a state-dependent force. We analyze the error sources in this scheme and obtain low infidelity.

1. Introduction

Ion trap quantum computation is actively studied and so far has shown many promising results [1,2]. Since implementing large-scale quantum computation is quite challenging, a more reachable goal for the near future is the realization of quantum simulation. Various quantum simulations with trapped ions have been proposed, for example, different models in condensed matter physics [3–8], quantum optics [9,10], cosmological models [11], quantum open systems [12] or Dirac particles [13,14]. Realizing such quantum simulations with 30–40 ions would already be immensely rewarding as these problems are difficult even for today's supercomputers.

While the elementary one and two-qubit quantum logic gates and a few qubits quantum algorithms have been experimentally demonstrated [15,16], the current challenge for ion trap quantum simulation and computation is scalability. The alternatives that are currently being investigated are anharmonic linear ion traps [17] for long equally spaced ion strings, ion-shuttling [18,19], two-dimensional microtrap arrays [20] and the planar Coulomb crystals in Penning [21,22] or rf [22] traps. Shuttling ions and two-dimensional microtrap arrays require the precise microfabrication and a high degree of controllability. Planar Coulomb crystal might be a simpler alternative to realize quantum simulations with a few tens or maybe even a few hundreds of ions [22–24]. In addition, the triangular lattice structure of planar Coulomb crystals could be exploited for some quantum simulations, such as frustrated spin models.

The two-qubit gate in planar Coulomb crystals can be implemented by the "axial" pushing gate proposed by Porras and Cirac [21]. The pushing gate implements the controlled phase-shift gate $(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + e^{i\vartheta}|11\rangle\langle 11|)$ [20,25]. The ions in the excited state are displaced from their equilibrium positions by the state-dependent force, and due to the Coulomb repulsion a state-dependent phase shift is achieved. The pushing gate does not require the cooling of the ions to the motional ground state, and is relatively fast as compared to other entangling gates [25]. The state-dependent force for implementing the pushing gate can be realized in different ways, for instance, by an optical dipole force. For example, the force could be induced by a walking wave [1,26]. The problem of the optical dipole forces is the large scattering that decreases the fidelity [27]. Alternatively, magnetic fields could also generate state-dependent forces that would allow for the realization of scattering-free pushing gates [28]. However, this would be difficult to implement in the case of planar Coulomb crystals because the magnetic force is generated by microcoils placed underneath each ion [28].

We propose a simpler implementation of the pushing gate acting on ions in a planar Coulomb crystal. The proposed scheme is the following: a flat-top, red-detuned from the resonant frequency of internal states of ions $|g\rangle \leftrightarrow |e\rangle$, and tightly focused laser is used. The spatial intensity profile of the beam creates AC Stark

* Corresponding author.
E-mail addresses: kitaoka@lyman.q.t.u-tokyo.ac.jp (M. Kitaoka), hasegawa@sys.t.u-tokyo.ac.jp (S. Hasegawa).
1 Present address: Advanced Science Institute, RIKEN, Wako-shi, Saitama 351-0198, Japan.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2009.06.019

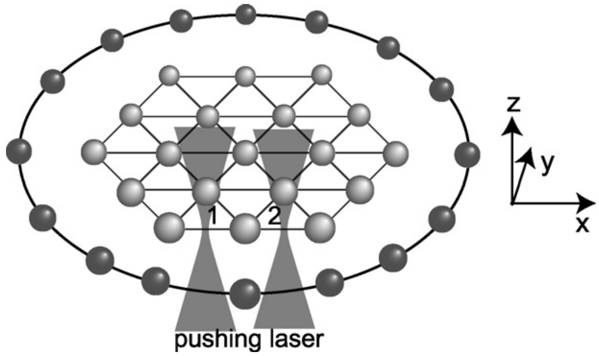

Fig. 1. The scheme of our proposed pushing gate in planar Coulomb crystals. The tightly-focused, red-detuned laser raises the spatial intensity profile and the trapped ions are pushed in the direction perpendicular to the crystal plane (axial direction).

shifts. The AC Stark shift forces the ions to be displaced towards the focusing point of the laser beam. Fig. 1 illustrates the idea of this scheme. We can realize the pushing force by controlling the shape of the laser beam, and obtain sufficient fidelity for quantum simulations. The ions in the outer shells are not utilized for quantum operations, but can be used for sympathetic cooling. In [29] we suggested using planar Coulomb bicrystals for efficiently cooling the operational ions.

The novelty of the proposal lies in the improved implementation scheme. Using a flat-top beam, rather than a walking wave, helps avoiding the disturbance of the in-plane vibrational modes which reduces the fidelity. Moreover, in the case of planar Coulomb crystals, the current scheme could be easier to realize experimentally. Numerical estimations using realistic experimental parameters demonstrate the feasibility of our proposal. The pushing-gate is important not only in the scalability of ion trap quantum computation using ion arrays, but also in realizing large-scale quantum simulations. The flexibility in its implementation is demonstrated by the various ways makes in which the pushing force can be produced (traveling wave, standing wave, walking wave, magnetic dipole force). Because the pushing gate will play a central role in the future developments of ion trap quantum computation and simulation, investigating novel alternative implementation schemes is useful for both theory and experiment.

The Letter is structured as following: first, we describe our proposed pushing gate theoretically and calculate the fidelity including the photon scattering, the nonuniformity of the laser beam and the movement of the ions. Then, we estimate the fidelity in the case of $^{40}Ca^+$ ions and discuss the application of the pushing gate to quantum simulations. Finally, we comment on the results and draw the conclusions.

2. Theoretical treatment

2.1. Pushing gate in planar Coulomb crystals

In our scheme, we assume that the ions have a ground state denoted by $|g\rangle$ and an excited state denoted by $|e\rangle$. The ground states are splitted into $|g(+1/2)\rangle$ and $|g(-1/2)\rangle$. We encode the states $|g(+1/2)\rangle$ and $|g(-1/2)\rangle$ into the logical states $|1\rangle$ and $|0\rangle$, respectively. When the laser irradiates the ions, the energy levels are shifted due to the AC Stark effects. If the laser frequency is red-detuned, the ions are driven towards the laser intensity maximum. In contrast, if the laser frequency is blue-detuned, the ions are pushed away from the intensity maximum. For realizing the pushing gate, we use a polarized laser beam coupling $|1\rangle$ $(=|g(+1/2)\rangle)$ to $|e\rangle$, which produces the state-dependent force (Fig. 2(a)).

We assume that the kinetic energy of the trapped ions obeys the Boltzmann distribution at temperature T, and the initial phase

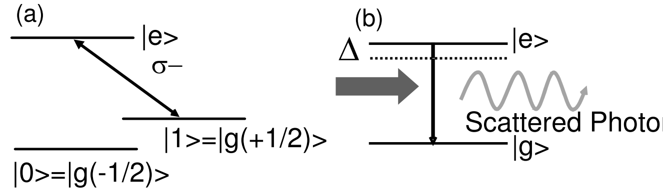

Fig. 2. (a) Ion energy diagram. The polarized laser couples the excited state $|e\rangle$ to the ground state $|1\rangle$. (b) Photon scattering. The population is pumped up to the excited state and then spontaneous emission occurs.

of the oscillation obeys a uniform distribution. Then, we can derive the average phase shift [25] due to the pushing force $F = \alpha\hbar\omega_z\xi(z)e^{-(t/\tau)^2}/a$ acting on the ions whose internal state is $|\alpha\rangle$ as

$$\vartheta = \theta\left[1 + \left(\frac{a}{d}\right)^2\left(\frac{3\xi(z)^2}{2} + \frac{9k_BT}{\hbar\omega_z}\right)\right], \tag{1}$$

where

$$\theta = \sqrt{\frac{\pi}{2}}\xi(z)^2\beta_z\omega_z\tau, \tag{2}$$

ω_z is the axial trapping frequency, $\xi(z)$ is the dimensionless force amplitude, τ is the gate duration, d is the mean distance between the equilibrium positions of the neighboring ions, $a = \sqrt{\hbar/m\omega_z}$ is the axial vibrational width of the motional ground state, $\beta_z = e^2/4\pi\epsilon_0 m\omega_z^2 d^3$ is the ratio of the Coulomb $(e^2/4\pi\epsilon_0 d)$ and axial potential $(m\omega_z^2 d^2/2)$ energies (the β parameter in the axial direction [21]), m is mass of the ions, and e is the elementary electric charge. To maximize the entanglement, meaning that the controlled-Z gate is realized, we choose that the final phase shift is to be π. In order to reduce the infidelity, the π-pulse method [25] is available. It uses a pair of π pulses to implement a “spin echo” to cancel correlated errors (see [25]). In the following, we consider the π-pulse method and choose τ such that $\theta = \pi/2$ (final phase shift is π). The β parameter [21] in the radial direction is

$$\beta_{xy} = \frac{2e^2}{4\pi\epsilon_0 m\omega_{xy}^2 d^3}. \tag{3}$$

The spectra of the phonon modes in planar Coulomb crystals are relatively complex. As a result of the displacement of the ions, the harmonic oscillations in the crystal are disturbed. In order to reduce this phenomenon, some special conditions are required. First, the potential energy should be larger than the Coulomb energy, i.e. the ions should be independent harmonic oscillators ($\beta_z \ll 1$). Secondly, the frequencies of the in-plane oscillation has to be smaller than the frequencies of the axial oscillation. We need to calculate the spectra of the phonons in order to check the fulfillment of these conditions. For the “axial” pushing gate, the displacement in the axial direction (perpendicular on the crystal plane) is required while the displacement in the x–y plane is not needed. Note that a Gaussian beam would drive the ions in the x–y plane. In order to avoid this, one should rather use a flat-top beam. With a flat-top beam, the AC Stark shift is raised uniformly and the ions are not displaced in the x–y plane.

2.2. Laser–ion interaction

The optical dipole potential of the state $|1\rangle$ is given by

$$U(z) = -\frac{\hbar|\Omega|^2}{4}\left(\frac{1}{\Delta} + \frac{1}{\Delta'}\right), \tag{4}$$

where $\Delta = \omega_0 - \omega_L$ and $\Delta' = \omega_0 + \omega_L$. The Rabi frequency Ω can be written as