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Freezing transition in bi-directional CA model for facing pedestrian traffic

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A B S T R A C T

We present a bi-directional cellular automaton (CA) model for facing traffic of pedestrians on a wide passage. The excluded-volume effect and bi-directionality of facing traffic are taken into account. The CA model is not stochastic but deterministic. We study the jamming and freezing transitions when pedestrian density increases. We show that the dynamical phase transitions occur at three stages with increasing density. There exist four traffic states: the free traffic, jammed traffic 1, jammed traffic 2, and frozen state. At the frozen state, all pedestrians stop by preventing from going ahead each other. At three transitions, the pedestrian flow changes from the free traffic through the jammed traffic 1 and jammed traffic 2, to the frozen state.

1. Introduction

Recently, pedestrian and vehicular traffic flows have attracted considerable attention [1–5]. Many observed dynamical phenomena in pedestrian and traffic flows have been successfully reproduced with physical methods. The pedestrian flow dynamics is closely connected with the self-driven many-particle system [6]. It has also encouraged physicists to study evacuation processes by self-driven many-particle models [7–13]. The pedestrian and vehicular traffic models have been applied to the traffic flow of such mechanical mobile objects as robots [14,15].

The typical pedestrian flows have been simulated by the use of a few models in two-dimensional space: the lattice-gas model of biased-random walkers [11–16], the molecular dynamic model of active walkers [6,10,17], and the cellular automaton model [7,8]. Their models are not deterministic but stochastic. Their models are described in two-dimensional space. The molecular dynamic model of active walkers is described by the behavioral (or generalized) force on two-dimensional off-lattice. The lattice-gas model of biased-random walkers and the CA model are described by stochastic rules on the square lattice. Helbing et al. have found that the "freezing by heating" occurs in the facing pedestrian traffic by the use of the molecular dynamic model of active walkers [17]. By using the lattice gas model of biased-random walkers, Muramatsu et al. have found independently that the freezing transition occurs from the free traffic to the frozen (stopping) state when the pedestrian density is higher than the critical value [16]. The freezing transition in the facing pedestrian traffic has been studied by some researchers [18,19].

In the jamming transition, pedestrian flow in the crowd changes from the free traffic to the jammed traffic in which pedestrians are distributed heterogeneously and move slowly. In the freezing transition, pedestrian flow change to the frozen state in which all pedestrians cannot move by preventing from going ahead each other. The analytical works are unknown for the facing pedestrian flow. The pedestrian flow has been investigated by the numerical simulation of the stochastic models on two-dimensional space. It is not easy to analyze the two-dimensional stochastic models because the dynamical behavior is complex. However, the one-dimensional deterministic CA models have not been proposed for facing traffic of pedestrians until now.

In this Letter, we present the one-dimensional, deterministic, and bi-directional CA model for the facing pedestrian traffic. We study the dynamical states and dynamical phase transitions in the model of facing pedestrian traffic. We show that there exist four pedestrian states and the jamming and freezing transitions occur when pedestrian density increases. We show that the bi-

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directional CA model reproduces the Burgers CA model of unidirectional multi-lane traffic in the limit of no facing pedestrians.

2. Bi-directional CA model

We consider the facing (bi-directional) traffic of pedestrians on a wide passage. There exist two kinds of walkers on the passage: the one is the walkers moving to the east and the other the walkers moving to the west. The walker moving to the east (or west) interacts highly with the other walkers in the front. When the density of walkers ahead is higher, the current decreases more because the movement of walkers will be prevented by other walkers.

We consider the one-dimensional approximation for two-dimensional facing pedestrian traffic. We approximate the facing traffic (counter flow) on two-dimensional lattice as that on one-dimensional lattice because walkers to east or to west move unidirectionally on the average. M walkers can exist on a cell (site) at its maximum. This means that the passage consists of M lanes for walkers. The width of passage is M.

We define the number that walkers to east (to west) exist on cell (site) i at time t as $N_E(i,t)$ $(N_W(i,t))$. The states of walkers to east are updated in parallel at every odd discrete time step, whereas those of walkers to west are updated in parallel at every even discrete time step. We apply the conservation law of walker number $N_E(i,t)$ $(N_W(i,t))$ to the facing traffic. The number $N_E(i,2t+1)$ of walkers to east existing on site i at time $2t+1$ is described by the following:

$$\begin{aligned} &N_E(i,2t+1)\\ &\quad = N_E(i,2t-1)\\ &\qquad + \min\bigl[N_E(i-1,2t-1), M - N_E(i,2t-1) - N_W(i,2t)\bigr]\\ &\qquad - \min\bigl[N_E(i,2t-1),\\ &\qquad M - N_E(i+1,2t-1) - N_W(i+1,2t)\bigr]. \end{aligned} \tag{1}$$

The number $N_W(i,2t+2)$ of walkers to west existing on site i at time $2t+2$ is described by the following:

$$\begin{aligned} &N_W(i,2t+2)\\ &\quad = N_W(i,2t)\\ &\qquad + \min\bigl[N_W(i+1,2t), M - N_E(i,2t+1) - N_W(i,2t)\bigr]\\ &\qquad - \min\bigl[N_W(i,2t), M - N_E(i-1,2t+1) - N_W(i-1,2t)\bigr], \end{aligned} \tag{2}$$

where $\min[A,B]$ is the minimal function: $\min[A,B] = A$ if $A < B$ and $\min[A,B] = B$ if $A > B$. The second term on the right hand in Eq. (1) represents the inflow of a walker to east from site $i-1$ to site i between $2t$ and $2t+1$. The third term represents the outflow of a walker to east from site i to site $i+1$ between $2t$ and $2t+1$. Similarly, the second and third terms of Eq. (2) represents the inflow and outflow of a walker to west on site i between $2t+1$ and $2t+2$.

The excluded-volume effect is taken into account via $\min[N_E(i-1,t), M - N_E(i,t) - N_W(i,t)]$. $M - N_E(i,t) - N_W(i,t)$ represents the unoccupied space at site i and time t. If $N_E(i-1,t) > (M - N_E(i,t) - N_W(i,t))$, $(M - N_E(i,t) - N_W(i,t))$ walkers of $N_E(i-1,t)$ walkers can move to the unoccupied space.

Eqs. (1) and (2) are a couple of nonlinear difference equations. It is not easy to obtain the analytical solution but possible to obtain the numerical solution.

In the limit of no walkers to west, Eq. (1) reduces to the Burgers CA model proposed by Nishinari and Takahashi [20]:

$$\begin{aligned} N_E(i,t+1) = N_E(i,t) &+ \min\bigl[N_E(i-1,t), M - N_E(i,t)\bigr]\\ &- \min\bigl[N_E(i,t), M - N_E(i+1,t)\bigr]. \end{aligned} \tag{3}$$

When $M = 1$ in Eq. (3), the Burgers CA model (3) reduces to the rule-184 CA.

3. Simulation and result

We carry out the numerical simulation for bi-directional CA model described by Eqs. (1) and (2). The boundaries are periodic. We consider the following initial condition. Walkers to east and to west distribute uniformly on the passage, respectively. The initial condition is described by

$$N_E(i,0) = N_{E,0} \quad \text{and} \quad N_W(i,0) = N_{W,0}. \tag{4}$$

The densities of walkers to east and to west are defined, respectively, as $\rho_E = N_{E,0}/M$ and $\rho_W = N_{W,0}/M$.

3.1. Current-density diagram

We study the dependence of current J_E/M of walkers to east on initial density $N_{E,0}/M$ of walkers to east under the initial condition of $N_{W,0} = \text{const}$.

If passage width M is sufficiently large, the facing traffic shows a sharp transition and the freezing transition depends little on M. Because the boundary is periodic, passage length L has little effect on the traffic when L is large. The width and length of the passage are set as $M = 200$ and $L = 100$.

We add a small perturbation to initial condition (4): $N_E(50,0) = N_{E,0} - 1$ and $N_E(60,0) = N_{E,0} + 1$. Fig. 1(a) shows the plots of current J_E/M against initial density $N_{E,0}/M$ under the initial condition of $N_{W,0} = 25$. Circles indicate the simulation result. The current increases linearly with density until point a, then decrease from point a to pint b, decrease abruptly and discontinuously at point b, and become zero at point c. Thus, the slope of current changes discontinuously at three transition points a, b, and c. The current becomes zero at transition point c. All walkers cannot move when the density is higher than ρ_c (the density at transition point c). The slopes of segment $a-b$ and segment $b'-c$ are -1.

We study the effect of the initial perturbation on the facing traffic. Fig. 1(b) shows the plots of current J_E/M against initial density $N_{E,0}/M$ for perturbation $N_E(50,0) = N_{E,0} - 1$ and $N_E(70,0) = N_{E,0} + 1$, where passage width $M = 200$, passage length $L = 100$, and $N_{W,0} = 25$. The density of transition point b is higher than that in Fig. 1(a). Also, the density of transition point c is higher than that in Fig. 1(a). Transition points b and c changes with the perturbation. Fig. 1(c) shows the plots of current J_E/M against initial density $N_{E,0}/M$ for no perturbations. The initial condition in the case of no perturbations is given by Eq. (4). The current increases linearly with density, reaches its maximum value at point a, and then decreases from point a with increasing density. The transition points b and c do not appear but the current-density diagram shows only one transition point a. The density of transition point a does not change with any perturbations.

When the density is less than that of transition point a, the current is given by

$$J_E = N_{E,0} \quad \text{for } N_{E,0}/M < \rho_a, \tag{5}$$

where ρ_a is the density at transition point a. When the density is higher than ρ_a, the current is given by

$$J_E = M - N_{E,0} - N_{W,0} \quad \text{for } N_{E,0}/M > \rho_a. \tag{6}$$

At the first transition point, Eq. (5) equals to Eq. (6).