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A new criterion for chaos and hyperchaos synchronization using linear feedback control

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Abstract

Based on the characteristic of the chaotic or hyperchaotic system and linear feedback control method, synchronization of the two identical chaotic or hyperchaotic systems with different initial conditions is studied. The range of the control parameter for synchronization is derived. Simulation results are provided to show the effectiveness of the proposed synchronization method.

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Keywords: Chaos synchronization; Hyperchaos synchronization; Linear feedback control; Largest Lyapunov exponent

1. Introduction

Synchronization of chaotic or hyperchaotic system has been studied with increasing interest in the last few years due to its potential technological applications, such as in the fields of secure communication, and so on. So, various effective methods which are applied to make two identical or different chaotic or hyperchaotic systems up to synchronization are proposed and confirmed since PC method [1] has been advanced. For example, linear feedback control [2], nonlinear feedback control [3,4], adaptive control [5], backstepping control [6,7] and so on.

Each method has its character and fits to its certain area. Among them, the linear feedback control is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration and implementation. However, many researchers who adopt this method to synchronize the two chaotic or hyperchaotic system are likely to design the appropriate Lyapunov function in order to compute the range of satisfied control parameter based on the Lyapunov stability theorem [8,9]. In a some sense, it has some more complex mathematical derivation. Furthermore, in some chaotic or hyperchaotic systems, it is too difficult to design the right Lyapunov function, especially in some piecewise chaotic or hyperchaotic systems. So, studying on some new approach to gain the range of satisfied control parameter will be important and valuable. In this Letter, based on the characteristic of the chaotic or hyperchaotic system, a new, simple and yet easily verified sufficient condition is established for chaos or hyperchaos synchronization, which is also applicable to a large class of generalized chaotic or hyperchaotic systems.

This Letter is organized as follows. In Section 2, based on the characteristic of chaotic or hyperchaotic system and linear feedback control theory, a rather simple conclusion of chaos or hyperchaos synchronization is derived for two chaotic or hyperchaotic systems with different initial conditions through a linear feedback controller. In Section 3, a brief description of the Chua's circuit is introduced and the approach is applied to this circuit. In Section 4, a brief description of the MCK's circuit is introduced and the approach is applied to this circuit. In Section 5, numerical simulations are given for illustration and verification. Finally, some concluding remarks and comments are given.

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2. The principle of linear feedback control

Let us consider the n-dimension autonomous chaotic or hyperchaotic system which is considered as a drive system:

$$\dot{X} = F(X). \tag{1}$$

The response system is given by:

$$\dot{Y} = F(Y). \tag{2}$$

The equations will be formed as follows when it is added the linear feedback controller to the response system

$$\begin{cases} \dot{X} = F(X), \\ \dot{Y} = F(Y) + K(X - Y). \end{cases} \tag{3}$$

When we define error vector as $E = X - Y$, Eq. (3) can be changed into the following form:

$$\begin{cases} \dot{X} = F(X), \\ \dot{E} = G(X, Y, K), \end{cases} \tag{4}$$

where $F = (f_1, f_2, \dots, f_n)^{\mathrm{T}}$, $G = (g_1, g_2, \dots, g_n)^{\mathrm{T}}$, $X = (x_1, x_2, \dots, x_n)^{\mathrm{T}}$, $Y = (y_1, y_2, \dots, y_n)^{\mathrm{T}}$, $E = (e_1, e_2, \dots, e_n)^{\mathrm{T}}$ are n-dimension vectors, $K = \mathrm{diag}[k_1, k_2, \dots, k_n]^{\mathrm{T}}$ is linear feedback control parameter vector. T is transposition. The largest Lyapunov exponent is supposed as $\lambda_{\max} = \max(\lambda_1, \lambda_2, \dots, \lambda_n)$ and the value of linear feedback control parameter is defined as $k = k_1 = \cdots = k_n$. The synchronization problem is to design the control parameter which synchronizes the states of both the drive and response system.

Theorem 1. *If we choose the linear feedback control parameter is greater than the largest Lyapunov exponent of chaotic or hyperchaotic system, viz., $k > \lambda_{\max}$, the two identical chaotic or hyperchaotic systems with different initial conditions will be synchronized by using linear feedback control.*

Proof. First, the vector field divergence of system (1) and system (2) are defined as

$$D(X) = \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i}, \qquad D(Y) = \sum_{i=1}^{n} \frac{\partial f_i}{\partial y_i}, \tag{5}$$

respectively, and the Lyapunov exponents are $\lambda_i(X)$, $\lambda_i(Y)$, respectively. Based on the relationship between the vector field divergence and the sum of the whole Lyapunov exponents of the system and system (2) is identical to system (1), the following equations can be obtained as [10]

$$D(X) = D(Y) = \sum_{i=1}^{n} \lambda_i(X) = \sum_{i=1}^{n} \lambda_i(Y). \tag{6}$$

For system (3)

$$D = D(X) + D(Y) - \sum_{i=1}^{n} k_i. \tag{7}$$

For system (4)

$$D = D(X) + D(E), \tag{8}$$

where $D(E) = \sum_{i=1}^{n} \lambda_i(E) = \sum_{i=1}^{n} \partial g_i / \partial e_i$ is the vector field divergence of the error dynamical system and $\lambda_i(E)$ is its Lyapunov exponent.

So, the new equation can be obtained as follows from (6), (7), (8)

$$D(E) = D(X) - \sum_{i=1}^{n} k_i, \tag{9}$$

when $D(E) < 0$, $D(X) - \sum_{i=1}^{n} k_i < 0$, viz., $k > \lambda_{\max}$, the state of the error dynamical system will be stable at the zero equilibrium point. So, synchronization of the two identical systems with different initial conditions will be achieved. □

3. Synchronization of two identical Chua's circuits

To verify the use of this new criterion for chaos synchronization, Chua's circuit is considered in the form of [11]

$$\begin{aligned} \dot{x}_1 &= \alpha\big(x_2 - x_1 - f(x_1)\big), \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -\beta x_2, \end{aligned} \tag{10}$$

and

$$f(x_1) = \begin{cases} bx_1 + a - b, & x_1 \geqslant 1, \\ ax_1, & |x_1| \leqslant 1, \\ bx_1 - a + b, & x_1 \leqslant -1, \end{cases} \tag{11}$$

where a, b, α, β are constants and x_1, x_2, x_3 are variables of Chua's circuit. Taking $\alpha = 10$, $\beta = 15.68$, $a = -1.2768$, $b = -0.6888$, an example of the two scroll chaotic attractors is obtained, as depicted in Figs. 1(a)–(b). The Lyapunov exponents of this chaotic system are obtained $\lambda_1 = 0.3816$, $\lambda_2 = 0.0000$, $\lambda_3 = -3.6772$. Apparently, $\lambda_{\max} = \max(\lambda_1, \lambda_2, \lambda_3) = \lambda_1$. When the linear feedback controller is added to the Chua's circuit, the following equations can be obtained as

$$\begin{aligned} \dot{y}_1 &= \alpha\big(y_2 - y_1 - f(y_1)\big) - k_1(y_1 - x_1), \\ \dot{y}_2 &= y_1 - y_2 + y_3 - k_2(y_2 - x_2), \\ \dot{y}_3 &= -\beta y_2 - k_3(y_3 - x_3), \end{aligned} \tag{12}$$

where $k = k_1 = k_2 = k_3$ and $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$ are defined. We subtract system (10) from system (12) and get the error equations as follows:

$$\begin{aligned} \dot{e}_1 &= \alpha\big(e_2 - e_1 - f(y_1) + f(x_1)\big) - ke_1, \\ \dot{e}_2 &= e_1 - e_2 + e_3 - ke_2, \\ \dot{e}_3 &= -\beta e_2 - ke_3. \end{aligned} \tag{13}$$

According to Theorem 1, let $k > 0.3816$ and the synchronization of two Chua's circuits will be realized.

4. Synchronization of two identical MCK's circuits

To verify the use of the new criterion for hyperchaos synchronization, MCK's circuit, as a drive system, is considered in