Physics Letters A 373 (2009) 2438–2444

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

On some applications of diffusion processes for image processing

S. Morfu

Laboratoire d'Electronique, Informatique et Image (LE2i), UMR Cnrs 5158, Aile des Sciences de l'Ingénieur, BP 47870, 21078 Dijon Cedex, France

ARTICLE INFO

Article history:
Received 25 March 2009
Accepted 15 April 2009
Available online 15 May 2009
Communicated by A.R. Bishop

PACS:
05.45.-a
89.20.-a
07.05.Pj

Keywords:
Cellular Nonlinear Network (CNN)
Image processing
Nonlinear dynamics

ABSTRACT

We propose a new algorithm inspired by the properties of diffusion processes for image filtering. We show that purely nonlinear diffusion processes ruled by Fisher equation allows contrast enhancement and noise filtering, but involves a blurry image. By contrast, anisotropic diffusion, described by Perona and Malik algorithm, allows noise filtering and preserves the edges. We show that combining the properties of anisotropic diffusion with those of nonlinear diffusion provides a better processing tool which enables noise filtering, contrast enhancement and edge preserving.

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1. Introduction

Since the past ten years, signal or image processing inspired by the properties of nonlinear systems has widely stimulated the interest of researchers. The main motivation to consider nonlinearity is to mimic the behavior of natural systems which are able to solve complex problems. Indeed, natural phenomena are highly nonlinear and ruled by nonlinear differential equations [1–3] which can also describe the voltage evolution in analog circuits [4,5] or the evolution of concentrations in chemical media [6]. It is in this context that unconventional methods of computing based on nonlinear differential equations have been proposed as a new direction of research with unsuspected applications [6]. For instance, the properties of photosensitive chemical media has allowed to implement complex tasks such as, controlling mobile robot [7–9], image skeletonisation [10] or extraction of individual components of an image where components overlap [11]. In fact, the efficiency of these reaction–diffusion media comes both from their highly parallel architecture and their nonlinear properties inherited from natural systems. These two features are also shared by the so-called Cellular Nonlinear Network (CNN) introduced by L. Chua in the late eighties as a novel class of nonlinear electronic network capable of processing information in real-time [4,12]. Among the processing tools developed with CNN, we can cite edge detection [5,12], extraction of image skeleton [13], color image processing [14], object oriented segmentation [15–17], contrast enhancement [18–20] or impulsive noise removal [21] (see [22], for an overview of the applications).

In the field of artificial intelligence, these last two filtering tasks often constitute pre-processing tools to develop more complex pattern recognition applications. Therefore, removing noise from a coherent information and contrast enhancement appear as two fundamental research directions in image processing. Concerning noise filtering, a classical algorithm based on the properties of anisotropic diffusion has been proposed in the early nineties by Perona and Malik [23]. Since its formulation, this algorithm has been widely used in a rich variety of image filtering fields [24], such as medical image analysis [25], astronomical image restoration [26], to cite but a few. However, this algorithm does not allow a strong contrast enhancement since it derives from a linear diffusion process which includes anisotropy.

By contrast, it has been shown that purely nonlinear isotropic diffusion processes, obeying to Fisher equation, can be used to perform contrast enhancement [5]. Unfortunately, owing to its isotropic properties, this nonlinear filter does not preserves the edges of the initial image.

Despite various papers devoted to the effects of inhomogeneities in purely nonlinear diffusive media [27–32], to our knowledge, there exist very few image processing applications which takes benefits of the anisotropic behaviour of these natural systems. In this Letter, considering the nonlinear Fisher equation, we investigate how nonlinearity and anisotropy can be combined to provide an efficient image processing tool. Especially, we present a new algorithm for noise filtering based on nonlinear and anisotropic diffusion processes which also performs contrast en-

E-mail address: smorfu@u-bourgogne.fr.

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doi:10.1016/j.physleta.2009.04.076

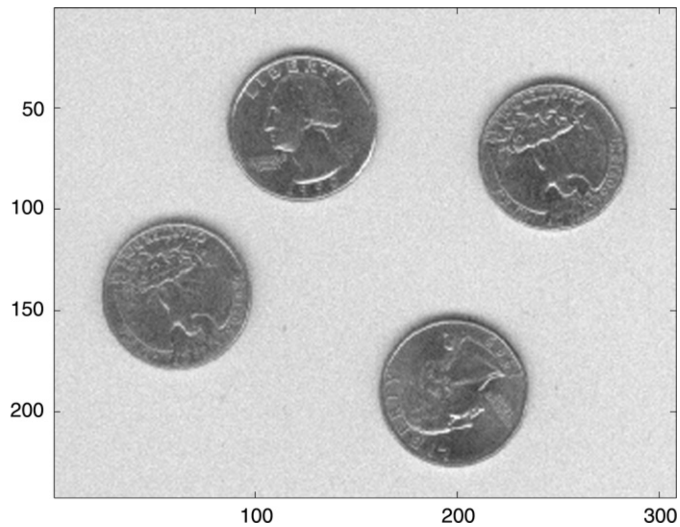

Fig. 1. Initial image to process.

hancement while the image edges are preserved. For a sake of clarity, we first propose an overview of the existing filtering tasks inspired by different diffusion processes. In particular, we review their advantages and drawbacks.

The Letter is organized as follow. In the second section, we present the linear isotropic diffusion process applied to the noisy image represented in Fig. 1. Section 3 is devoted to the presentation of the nonlinear isotropic process. In the next section, the results obtained with the standard Perona and Malik algorithm are then briefly presented. Then, we introduce the new algorithm which takes advantage of the filters presented in the previous sections. In the last section, we finally provide some concluding remarks.

2. Linear diffusion process

In this section, we consider a linear diffusion process ruled by the heat equation

$$\frac{\partial X(x,y,t)}{\partial t} = D\Delta(X), \tag{1}$$

where X represents the temperature at a given point in space of coordinates (x, y), D the diffusion coefficient and Δ the Laplace operator. Since D does not depend on the spatial coordinates, the diffusion process is isotropic. In the context of image processing, X denotes the brightness of a continuous image. Owing to the discrete nature of image, the continuous equation (1) is discretized in space, but also in time with a time step dt as follow:

$$X_{i,j}^{t+dt} = X_{i,j}^{t} + \frac{dt}{|N_r|} D \sum_{(k,l)\in N_r} \left(X_{k,l}^{t} - X_{i,j}^{t}\right)$$

$$\text{with } i, j = 2, 3, \ldots, N-1, 2, 3, \ldots, M-1. \tag{2}$$

In Eq. (2), $N \times M$ corresponds to the image size and $|N_r|$ represents the number of neighbors, that is 4 except at the image boundaries. In addition, the set of the 4 neighbors is $Nr = \{(i-1, j), (i+1, j), (i, j+1), (i, j-1)\}$. Moreover, $X_{i,j}^{t}$ denotes the gray level of the filtered image at the time step t, while the gray level at the next time step is $X_{i,j}^{t+dt}$. Lastly, the initial conditions applied to the set of equations (2) correspond to the gray levels of the noisy image of Fig. 1. To complete the description, we define the following neighborhood for the 4 edges of the image

$$\begin{aligned}
&Nr = \{(i-1,1), (i+1,1), (i,2)\}, \quad \text{for } i = 2, 3N-1,\\
&Nr = \{(i-1,M), (i+1,M), (i,M-1)\}, \quad \text{for } i = 2,3,\ldots,N-1,\\
&Nr = \{(1,j-1), (1,j+1), (2,j)\}, \quad \text{for } j = 2,3,\ldots,M-1,\\
&Nr = \{(N,j-1), (N,j+1), (N-1,j)\}, \quad \text{for } j = 2,3,\ldots,M-1,
\end{aligned} \tag{3}$$

while for the image corners, we consider the two nearest neighbors, that is

$$\begin{aligned}
&Nr = \{(2,1), (1,2)\}, \quad \text{for } (i,j) = (1,1),\\
&Nr = \{(N,2), (N-1,1)\}, \quad \text{for } (i,j) = (N,1),\\
&Nr = \{(1,M-1), (2,M)\}, \quad \text{for } (i,j) = (1,M),\\
&Nr = \{(N,M-1), (N-1,M)\}, \quad \text{for } (i,j) = (N,M).
\end{aligned} \tag{4}$$

The filtered images for different processing times $t = 2$ and $t = 6$, are then obtained numerically by integrating the algorithm (2) with the boundary conditions (3)–(4). The resulting images are reported in Fig. 2. Moreover, the profile of the line 50 of the image is also represented for different processing times in order to characterize the effects of the linear diffusion process. This profile reveals that initially (for $t = 0$), the image is slightly noisy. As the processing time increases, a low-pass filter behaviour is observed since the noise completely disappears after a processing time $t = 2$. However, the edges of the coins of the image are less and less localized and the details of the image are also removed by this linear and isotropic process. Therefore, the filtered image becomes blur. Lastly, we note that the profiles presented in Fig. 2(c) show that the contrast of the initial image is never enhanced.

3. Nonlinear isotropic diffusion process

In this section, we consider the Fisher equation which describes the transport mechanism in living cells [1], but which also allows to perform contrast enhancement and image segmentation [5,17]. This equation in its continuous form writes:

$$\frac{\partial X(x,y,t)}{\partial t} = D\Delta(X) + f(X), \tag{5}$$

where D is a diffusion coefficient and $f(X)$ represents a nonlinearity, commonly chosen cubic, that is:

$$f(x) = -\beta x(x-a)(x-1). \tag{6}$$

In Eq. (6), β adjusts the weight of the nonlinearity since for $\beta = 0$, Eq. (5) tends to Eq. (1) and thus the process becomes linear. Moreover, a is called threshold of the nonlinearity and is set to $a = 1/2$ to ensure the symmetry of the nonlinearity [5]. To realize image processing, a discretization of Eq. (5) with time step dt leads to

$$X_{i,j}^{t+dt} = X_{i,j}^{t} + \frac{dt}{|N_r|} D \sum_{(k,l)\in N_r} \left(X_{k,l}^{t} - X_{i,j}^{t}\right) + dt\, f\left(X_{i,j}^{t}\right). \tag{7}$$

This nonlinear filter has been numerically implemented. The results are summarized in Fig. 3, where the filtered image for $t = 6$ shows that the blur effect is still present. Indeed, the continuous equation is isotropic since the diffusion coefficient D remains constant versus the space coordinate x and y. Therefore, the edges loose their sharp profile. However, as exhibited in Fig. 3(c) for the processing time $t = 6$, the amplitudes of the profile of the filtered image exceed those of Fig. 2(c), conveying the fact that the contrast is enhanced compared to a linear process. This effect is mainly imputable to the properties of the nonlinear systems which tends to evolve towards its steady states defined by the roots of the cubic nonlinearity, namely 0 (black) and 1 (white) [17].

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