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A new dynamic model for heterogeneous traffic flow

T.Q. Tang [a,b,*], H.J. Huang [b], S.G. Zhao [c], H.Y. Shang [d]

[a] School of Transportation Science and Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China
[b] School of Economics and Management, Beijing University of Aeronautics and Astronautics, Beijing 100191, China
[c] School of Aeronautic Science and Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China
[d] Institute of Transportation Engineering, Tsinghua University, Beijing 10084, China

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ABSTRACT

Based on the property of heterogeneous traffic flow, we in this Letter present a new car-following model. Applying the relationship between the micro and macro variables, a new dynamic model for heterogeneous traffic flow is obtained. The fundamental diagram and the jam density of the heterogeneous traffic flow consisting of bus and car are studied under three different conditions: (1) without any restrictions, (2) under the action of the traffic control policy that restrains some private cars and (3) using bus to replace the private cars restrained by the traffic control policy. The numerical results show that our model can describe some qualitative properties of the heterogeneous traffic flow consisting of bus and car, which verifies that our model is reasonable.

1. Introduction

So far, many traffic models have been developed to study the complex traffic phenomena and many important results have been obtained [1–16]. However, these models cannot directly be used to study heterogeneous traffic flow since various heterogeneous factors (e.g. vehicle heterogeneity, speed heterogeneity, driver heterogeneity, etc.) were not considered. Later, scholars developed one extended Lighthill–Whitham–Richards (LWR) model [17–21] to study heterogeneous traffic flow, but the extended model cannot describe various non-equilibrium properties of heterogeneous traffic flow since the speed of each class cannot deviate from its equilibrium speed. Bagnerini and Rascle [22] adopted ordinary differential equation (ODE) to study the dynamic properties of heterogeneous traffic flow, but the properties are still presented in almost all micro-simulation packages. Gupta and Katiyar [23] presented a high order model for heterogeneous traffic flow, but each class will produce backward motion under some given condition [24]. Tang et al. [25] used the SG model [7,8] to study heterogeneous traffic flow and obtained an improved high order model for heterogeneous traffic flow, but this model is short of physical meaning.

Some models have been used to study heterogeneous traffic flow, but they cannot completely describe heterogeneous traffic flow. A new car-following model for heterogeneous traffic flow is presented in Section 2. Applying the relationship between the micro and macro variables [26], a new dynamic model for heterogeneous traffic flow is obtained in Section 3. Some numerical tests are carried out in Section 4. Finally, some qualitative conclusions are obtained in Section 5.

2. The car-following model

In general, the single-lane car-following model can be written as follows [1]:

$$\frac{d^2 x_n}{dt^2} = f(v_n, \Delta x_n, \Delta v_n), \tag{1}$$

where f is the stimulus function, x_n and v_n respectively the position and speed of the nth vehicle, $\Delta x_n = x_{n+1} - x_n$ the headway, $\Delta v_n = v_{n+1} - v_n$ the relative speed. Eq. (1) states that the acceleration of the nth vehicle is determined by the speed v_n, the headway Δx_n and the relative speed Δv_n. In order to improve the stability of traffic flow, scholars later developed the following improved car-following models [27–31]:

$$\frac{d^2 x_n}{dt^2} = f(v_n, \Delta x_n, \Delta x_{n+1}, \ldots, \Delta x_{n+m}, \Delta v_n), \tag{2}$$

where $\Delta x_{n+i} = x_{n+i+1} - x_{n+i}$. Zhao and Gao [32] found that a collision will occur under certain given conditions when using the full velocity difference (FVD) model [6] to describe traffic flow, then they proposed a new car-following model, i.e.,

$$\frac{d^2 x_n}{dt^2} = f\left(v_n, \Delta x_n, \Delta v_n, \frac{d^2 x_{n+1}}{dt^2}\right). \tag{3}$$

* Corresponding author at: School of Transportation Science and Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China.
E-mail address: tieqiaotang@buaa.edu.cn (T.Q. Tang).

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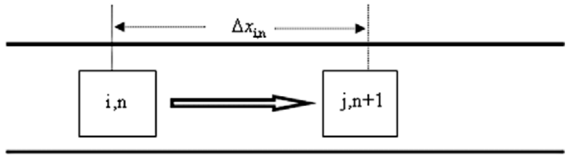

Fig. 1. Scheme of the car-following model.

To further enhance the stability of traffic flow, Wang et al. [33] proposed a multiple speed difference model as follows:

$$\frac{d^2 x_n}{dt^2} = f(v_n, \Delta x_n, \Delta v_n, \Delta v_{n+1}, \ldots, \Delta v_{n+k}), \tag{4}$$

where $\Delta v_{n+i} = v_{n+i+1} - v_{n+i}$.

The above car-following models can describe some complex phenomena, but they cannot be directly used to study heterogeneous traffic flow because they did not consider the effects that the probability of each class has on the car-following behavior of heterogeneous traffic flow. Fig. 1 is the car-following scheme of heterogeneous traffic flow, where i, n and $j, n+1$ state that the nth and $(n+1)$th vehicles are, respectively, the i-class and j-class, and the probability that the leading vehicle of the vehicle i, n is the j-class is p_{ij}. Note: we cannot here give the exact definition of the probability p_{ij} since it is often related to the structure of the heterogeneous traffic flow. The probability p_{ij} shows that the leading vehicle of the vehicle i, n is one random variable, so the headway and relative speed of the vehicle i, n are both random variables. Based on the property of random variable, we can find that the headway of the vehicle i, n will be equal to the mean of the random variables $x_{j,n+1} - x_{i,n}$ and that the effect of the relative speed on the vehicle i, n will be equal to the mean of the random variables $\lambda_{ij}(v_{j,n+1} - v_{i,n})$. Thus the acceleration of the nth vehicle can be reduced as follows:

$$\frac{dv_{i,n}(t)}{dt} = \kappa_i\left[V_{i,n}(\Delta x_{i,n}) - v_{i,n}\right] + \sum_{j=1}^{N} \lambda_{ij} p_{ij}(v_{j,n+1} - v_{i,n}), \quad j = 1, 2, \ldots, N, \tag{5}$$

where $v_{i,n}, \Delta x_{i,n} = \sum_{j=1}^{N} p_{ij}(x_{j,n+1} - x_{i,n})$ are, respectively, the speed and headway of the vehicle i, n, κ_i, λ_{ij} are reactive coefficients, N is the number of the classes, $V_{i,n}(\Delta x_{i,n})$ is the optimal speed of the vehicle i, n. Eq. (5) shows that the speed, the spacing and the reactive coefficients of each class have effects on the acceleration of the vehicle i, n, so it can better describe heterogeneous traffic flow. However, Eq. (5) does not consider the driver heterogeneity, so all the parameters in this Letter are irrelevant to drivers.

3. The dynamic model

In order to describe the dynamic property of heterogeneous traffic flow, we should adopt the method [26] to transform the micro variables in Eq. (5) into the macro ones, i.e.,

$$V_{i,n}\big(\Delta x_{i,n}(t)\big) \to v_{ie}(\rho), \qquad v_{i,n}(t) \to v_i(x,t),$$
$$v_{j,n+1}(t) \to v_j(x + \Delta_j, t),$$
$$\kappa_i \to \frac{1}{T_i}, \qquad \lambda_{ij} \to \frac{1}{\tau_{ij}}, \qquad p_{ij} \to p_j(x + \Delta_j, t), \tag{6}$$

where $v_{ie}(\rho)$ is the equilibrium speed of the i-class ($\rho = \sum_{i=1}^{N} \rho_i$ is the total density, where ρ_i is the i-class density); Δ_j is the distance between the leading and following vehicles; $v_i(x,t)$ is the i-class speed at the point (x,t); T_i, τ_{ij} are reactive coefficients; $p_j(x+\Delta_j, t)$ is the proportion of the j-class at the point $(x+\Delta_j, t)$. $p_j(x + \Delta_j, t)$ is complex and related to the j-class density at the point $(x+\Delta_j, t)$, so we assume $p_j(x+\Delta_j, t) = p_j(x,t)$ for simplicity. Thus, Eq. (5) can be rewritten as follows:

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{v_{ie}(\rho) - v_i}{T_i} + \sum_{j=1}^{N} \frac{1}{\tau_{ij}} p_j(x,t)\big(v_j(x + \Delta_j, t) - v_i(x,t)\big). \tag{7}$$

In order to expand Eq. (7) by Taylor series, we should rewrite it as follows:

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{v_{ie}(\rho) - v_i}{T_i} + \sum_{j=1}^{N} \frac{1}{\tau_{ij}} p_j(x,t)\big(v_j(x + \Delta_j, t) - v_j(x,t) + v_j(x,t) - v_i(x,t)\big). \tag{8}$$

Expanding Eq. (8) by Taylor series and neglecting the nonlinear terms, we have

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{v_{ie}(\rho) - v_i}{T_i} + \sum_{j=1}^{N}\left[c_{ij} p_j \frac{\partial v_j}{\partial x} + \frac{p_j}{\tau_{ij}}(v_j - v_i)\right], \tag{9}$$

where $c_{ij} = \Delta_j/\tau_{ij}$ is the propagating speed of the j-class's small perturbation resulted by the i-class. Fast vehicle responds to stimulus more quickly than slow one, so the small perturbation of fast vehicle will propagate faster than that of slow one (i.e. $c_{im} > c_{in}$ if $v_{mf} > v_{nf}$, where v_{mf} is the free flow speed of the m-class). The parameter c_{ij} has little qualitative effects on our results and we here mainly focus on studying some qualitative phenomena of heterogeneous traffic flow, so we can set c_{ij} as constant for simplicity (i.e. $c_{ij} = c_i$). In fact, the parameter c_{ij} will have great effects on the quantitative properties of the heterogeneous traffic flow, so we should in the future use many observed data to further calibrate this parameter.

Combining with conservation equation, we obtain a new dynamic model for heterogeneous traffic flow, i.e.

$$\frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} = F, \tag{10}$$

where

$$U = \begin{pmatrix} \rho_1 \\ v_1 \\ \rho_2 \\ v_2 \\ \cdots \\ \rho_N \\ v_N \end{pmatrix},$$

$$A = \begin{pmatrix} v_1 & \rho_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & v_1 - p_1 c_1 & 0 & -p_2 c_2 & \cdots & 0 & -p_N c_N \\ 0 & 0 & v_2 & \rho_2 & \cdots & 0 & 0 \\ 0 & -p_1 c_1 & 0 & v_2 - p_2 c_2 & \cdots & 0 & -p_N c_N \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & v_N & \rho_N \\ 0 & -p_1 c_1 & 0 & -p_2 c_2 & \cdots & 0 & v_N - p_N c_N \end{pmatrix},$$

$$F = \begin{pmatrix} 0 \\ \frac{v_{1e}(\rho) - v_1}{T_1} + \sum_{j=1}^{N} \frac{p_j}{\tau_{1j}}(v_j - v_1) \\ 0 \\ \frac{v_{2e}(\rho) - v_2}{T_2} + \sum_{j=1}^{N} \frac{p_j}{\tau_{2j}}(v_j - v_2) \\ \cdots \\ 0 \\ \frac{v_{Ne}(\rho) - v_N}{T_N} + \sum_{j=1}^{N} \frac{p_j}{\tau_{Nj}}(v_j - v_N) \end{pmatrix}.$$

Note that Eq. (10) is just the speed gradient model [7,8] if N is equal to 1. Since the proportion $p_j(x,t)$ is complex and related to