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Dynamical instability of a boson–fermion mixture at low dimensions

Z. Akdeniz [a], P. Vignolo [b,*]

[a] Piri Reis University, 34940 Tuzla-Istanbul, Turkey
[b] Institut Non Linéaire de Nice, Université de Nice-Sophia Antipolis, CNRS, 1361 route des Lucioles, 06560 Valbonne, France

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ABSTRACT

We examine theoretically the dynamical response of a homogeneous mixture of condensed bosons and spin-polarized fermions confined inside a quasi-two-dimensional or a quasi-one-dimensional geometry, considering quasi-three-dimensional boson–boson and boson–fermion interactions. We focus on the effects of low dimensions on the density response functions in the crossover from weak to strong boson–fermion coupling up to the onset of instability. The dynamical condition is found to be in agreement with a linear stability analysis at equilibrium.

1. Introduction

Fermionic atomic gases were brought together with bosonic atoms to quantum degeneracy in a ^{7}Li–^{6}Li mixture [1,2], ^{23}Na–^{6}Li mixture [3], and ^{87}Rb–^{40}K mixture [4–6]. The boson–fermion (BF) coupling strongly affects the equilibrium properties of the mixture and can drive quantum phase transitions, as predicted in several theoretical studies (for a review see [7]) and as recently observed in the context of three-dimensional (3D) atomic fermion–molecular boson mixtures [8,9], where the strong interspecies interaction leads to phase separation. Such mixtures can be realized from an imbalanced two-component Fermi gas (^{40}K–^{40}K or ^{6}Li–^{6}Li mixtures) where all minority fermions become bound as bosons and form a Bose–Einstein condensate (BEC). The Feshbach resonance exploited to drive the fermion–fermion interactions towards the formation of molecules determines the effective dimer–dimer and dimer–fermion scattering lengths, with a resulting strongly BF repulsion.

Though an imbalanced Fermi gas is a practical system with which to create a strongly repulsive BF mixture, the advantage of using two atomic species is that boson–boson and BF interactions can be driven independently and that one can access attractive BF interactions [10,11].

The stability condition for spatial demixing (or collapse) for a BF mixture depends on the Fermi energy [12] and thus on the geometry of the system [13]. On approaching a quasi-two-dimensional geometry (Q2D), in a pancake-shaped trap, the Fermi energy increases linearly with the number of fermions, so that the stability condition of the mixture becomes independent of the fermion density and involves only the scattering length and the transverse width of the cloud [14]. In a cigar-shaped trap, in the quasi-one-dimensional (Q1D) limit, the Fermi energy is proportional to the square of the number of fermions per unit length, with the consequence that the mixture becomes unstable at low linear fermion densities [15,16].

The dynamical properties of BF mixtures have been previously investigated theorically, mainly in the mixed phase, both for homogeneous systems [17–19] and in harmonically confined clouds. In the latter case these studies have exploited a sum-rule approach [20,21], perturbation theory [22], or a random-phase approximation (RPA) [23–25]. The spectrum behaviour approaching the instability have been studied in 3D [26,27] and in 1D [15], the dynamical signature of the approaching transition being a collective mode softening.

In this Letter we investigate the effect of the geometry on the spectrum of collective excitations of a zero-temperature BF mixture as the mixture approaches an instability. We adopt a RPA scheme and we examine a Q2D and a Q1D geometry in the limit where the effect of the lack confinement can be neglected. In analogy to what has already been shown, in both 3D [27] and in 1D [15],

* Corresponding author.
E-mail address: patrizia.vignolo@inln.cnrs.fr (P. Vignolo).

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for a pancake-shaped and for a cigar-shaped geometry, the arising instability is indicated by the softening of a hybridized mode. This dynamical condition is found to be in agreement with the linear stability analysis at equilibrium.

The Letter is organized as follows: in Section 2 we introduce the model and review the conditions for demixing and collapse, as obtained from static considerations. In Section 3 we introduce the RPA formalism and discuss the dynamical response in Q2D and in Q1D. Finally, Section 4 contains a summary and our main conclusions.

2. The model

We consider a homogeneous mixture of N_b hard-core bosons of mass m_b and N_f spin-polarized fermions of mass m_f at $T = 0$ K. We focus on two trap geometries: a disk geometry and a cylindric configuration with a tight confinement of frequency ω_0 in the azimuthal direction or in the radial plane respectively. If the trapping potential $\hbar\omega_0$ exceeds the ground state energy of the bosons and of the fermions, we have an effective low dimensional (LD) system. The collisions in such a LD system can be considered (i) strictly LD if the thickness $l \sim (\hbar/m_{b,f}\omega_0)^{1/2}$ of the disk (or of the cylinder) is smaller than the modulus of the boson–boson and boson–fermion scattering lengths a_{bb} and a_{bf}; (ii) quasi-LD if $l \sim a_{bb}, |a_{bf}|$; and (iii) quasi-3D if $l \gg a_{bb}, |a_{bf}|$. In this Letter we will focus on a mixture in reduced dimensionality with quasi-3D boson–boson and boson–fermion interactions. In this regime, the effective LD coupling strengths can be obtained by integrating over the frozen coordinates and read

$$g_{bb} = \frac{4\pi\hbar^2 a_{bb}}{m_b(\sqrt{2\pi}l)^L}, \qquad g_{bf} = \frac{2\pi\hbar^2 a_{bf}}{m_r(\sqrt{2\pi}l)^L} \tag{1}$$

with $m_r = m_b m_f/(m_b + m_f)$ being the reduced mass.

The equilibrium properties of the mixture can be studied by using a Thomas–Fermi approximation for the condensed bosons and for the spin-polarized fermions. The Thomas–Fermi approximation for the bosons assumes that the number of condensed (quasi-condensed) bosons is large enough that the kinetic energy term in the Gross–Pitaevskii equation may be neglected. It yields

$$g_{bb}n_b + g_{bf}n_f = \mu_b, \tag{2}$$

where n_b and n_f are the boson and fermion densities and μ_b the boson chemical potential. The Thomas–Fermi approximation for the spin-polarized fermions reads

$$\frac{1}{2m_f}\left(\frac{n_f}{A_L}\right)^{L/2} + g_{bf}n_b = \mu_f, \tag{3}$$

where $A_1 = 1/(\pi\hbar)$, $A_2 = 1/(4\pi\hbar^2)$ and μ_f the chemical potential for the fermions.

As the boson–fermion coupling increases, the mixture can become unstable against demixing (in the case $g_{bf} > 0$) or against collapse (in the case $g_{bf} < 0$). In a macroscopic system at given particle densities the linear stability analysis, $\det(\partial\mu_i/\partial n_j) \geqslant 0$, based on a mean-field energy functional predicts that the locations for demixing and collapse coincide and that the mixed gaseous cloud is stable if the condition

$$g_{bb}g_{ff} - (g_{bf})^2 \geqslant 0, \tag{4}$$

is fulfilled, with g_{ff} playing the role of an effective fermion–fermion repulsion due to the Pauli pressure in the Fermi gas in LD. In 3D $g_{ff} = \pi\hbar^2(4\pi/3n_f)^{1/3}$ and the mixed state is stable at given boson–fermion attractive or repulsive coupling if the fermion density is *below* a threshold [12]. In 2D $g_{ff} = 2\pi\hbar^2$ and the mixed state is either stable or unstable *regardless of the fermion areal density* [14]. In 1D $g_{ff} = \pi^2\hbar^2 n_f/m_f$ increases with n_f and the mixed state is stable when the linear density of fermions is *above* a threshold [15,16].

The stability condition given in Eq. (4) can be re-written

$$|a_{bf}| \leqslant |a^c_{bf}| = a_{bb}\left(\frac{2\pi m_r^2}{m_b m_f}\sqrt{\frac{2}{\pi}}\frac{l}{a_{bb}}\right)^{1/2} \tag{5}$$

in the disk geometry, and

$$|a_{bf}| \leqslant |a^c_{bf}| = a_{bb}\left(\frac{2\pi m_r^2}{m_b m_f}\frac{k_f^{1D} l^2}{a_{bb}}\right)^{1/2}, \tag{6}$$

in the cylinder geometry, with $k_f^{1D} = \pi n_f$ being the 1D Fermi wave number.

Eqs. (6) and (5) well depict the stability condition in the pancake-shaped [14] and in the cigar-shaped cloud [16] too, provided that the densities vary smoothly. In the cigar geometry, k_f^{1D} must be considered as the 1D Fermi wave number at the center of the cloud.

3. Dynamic response

We now consider the dynamical properties of the mixture. In the dilute limit, we can use a RPA approach which neglects correlations between density fluctuations, but satisfies the f-sum rules. The RPA yields the spectrum of collective excitations in the linear regime from a set of coupled equations for the density fluctuations δn_f and δn_b, which are obtained by assuming that the fluid responds as an ideal gas to external perturbing fields δU_f and δU_b plus fluctuations of the interaction energy (Hartree–Fock). The RPA equations in Fourier transform with respect to the time variable read

$$\delta n_f(\mathbf{r},\omega) = \int d^3r'\, \chi^0_{ff}(\mathbf{r},\mathbf{r}',\omega)\left[\delta U_f(\mathbf{r}',\omega) + g_{bf}\delta n_b(\mathbf{r}',\omega)\right] \tag{7}$$

and

$$\delta n_b(\mathbf{r},\omega) = \int d^3r'\, \chi^{\mathrm{Bog}}(\mathbf{r},\mathbf{r}',\omega)\left[\delta U_b(\mathbf{r}',\omega) + g_{bf}\delta n_f(\mathbf{r}',\omega)\right], \tag{8}$$

where χ^0_{ff} is the Lindhard density–density response function

$$\chi^0_{ff}(\mathbf{k},\omega) = \sum_{\mathbf{p}} \frac{f(\varepsilon_{\mathbf{p}}) - f(\varepsilon_{\mathbf{p+k}})}{\omega - (\varepsilon_{\mathbf{p+k}} - \varepsilon_{\mathbf{p}}) + i\eta} \tag{9}$$

with $\varepsilon_{\mathbf{k}} = \hbar k^2/2m_f$ and $f(\varepsilon) = \theta(\mu_f - \varepsilon)$. In the Bogolubov approximation for bosons the response function takes into account the boson–boson interactions, and has an algebraic form that does not depend on the dimensionality:

$$\chi^{\mathrm{Bog}} = \frac{n_b k^2/m_b}{\omega(\omega + i\eta) - c_b k^2 - (\hbar k^2/2m_b)^2}, \tag{10}$$

with $c_b = (g_{bb}n_b/m_b)^{1/2}$ the Bogolubov sound velocity. The system of Eqs. (7) and (8) can be rearranged as

$$\begin{pmatrix}\delta n_f\\ \delta n_b\end{pmatrix} = \begin{pmatrix}\chi_{ff} & \chi_{fb}\\ \chi_{bf} & \chi_{bb}\end{pmatrix}\begin{pmatrix}\delta U_f\\ \delta U_b\end{pmatrix} \tag{11}$$

with $\chi_{ff} = \chi^0_{ff}/(1 - g^2_{bf}\chi^{\mathrm{Bog}}\chi^0_{ff})$, $\chi_{bb} = \chi^{\mathrm{Bog}}/(1 - g^2_{bf}\chi^{\mathrm{Bog}}\chi^0_{ff})$ and $\chi_{fb} = \chi_{bf} = g_{bf}\chi^{\mathrm{Bog}}\chi^0_{ff}/(1 - g^2_{bf}\chi^{\mathrm{Bog}}\chi^0_{ff})$.

The density response functions of the mixture were first introduced in 3D by Yip [17], who evaluated numerically the spectra in the case of weak boson–fermion coupling, and have been used in the strong coupling regime to study the stability condition in 3D [26] and in 1D [15]. The aim of this Letter is to focus on the effects of low dimensions on the density response functions in the