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Induced N^2-cooperative phenomenon in an ensemble of the nonlinear coupled oscillators

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A B S T R A C T

In the article the cooperative N^2-effect is considered, that is the radiation whose power is $\sim N^2$, where N is the number of emitters which in this case is equal to the number of nonlinear coupled oscillators. They model the electrons moving in a semiconductor structure with grating (micro-undulator). The suggested effect is in a sense similar to Dicke superradiance, however it is not the spontaneous phase coherence arising in the ensemble of two-level atoms interacting via the emitted electromagnetic field, but rather, the result of interplay of two effects. The first one is the 'pumping wave' acting on the electrons and which is the result of undulator field, while the second is the backward effect of radiation which is produced by electrons moving within such micro-undulator. As a result, the specific phase coherence ('synchronization') develops in the ensemble of emitters and they start to generate as a single oscillating charge Ne, while the power of emitted radiation becomes $\sim N^2$. It is very probable, that the effect can be used for the developing of a new semiconductor-based room temperature source of the GHz and THz-radiation.

1. Introduction

The super-radiance is a cooperative phenomenon, radiation occurring due to spontaneous emission in an inverted system of interacting and initially independent two-level atoms (or more generally, emitters or oscillators). It was discovered by R. Dicke long ago in his seminal work [1] and since then intensively discussed in scientific literature [2–6]. Being in super-radiance state, the ensemble composed of a great number of atoms N, which are in excited state, emits radiation during the time τ_s much smaller then τ_0, the time of spontaneous decay of excited state of an individual atom. The effect results in the radiation intensity proportional to N^2, where N is the number of emitters, and it is due to specific phase correlations arising in the ensemble of radiating atoms.

As a matter of fact, the proportionality of radiation intensity to N^2 is a characteristic feature not only of the Dicke-model, since in many cases the initial correlations in the ensemble of oscillators can lead to this effect. Indeed, the possibility to get the generated power N^2 was also intensively discussed and in different contexts not in any way related to super-radiance discovered by Dicke. For example, the similar problem appears when people discuss the operation modes of free electron lasers (FEL). It was suggested many years ago that the specific phase coherence which leads to N^2-effect can be achieved, if the linear sizes l of the electron bunch in FEL is much smaller than the emitted wavelength λ. It is intuitively quite clear and seems to be almost obvious that if $l \ll \lambda$, the electron bunch generates waves as a single point source, that is, all electrons in a bunch generate waves of the same phase, and since the total charge of a bunch is equal Ne, the generated power is N^2. But if FEL is to generate visible light, one has to create a bunch whose linear size is about wavelength which is, mildly speaking, not a trivial task.

On the other hand, it is also known that Dicke super-radiance can occur not necessarily in the ensemble of two-level "atoms" (quantum oscillators), but in the ensemble of classical oscillators as well (see for example, the nice review [7]). These oscillators are considered as identical except the initial phases, which are or can be distributed at random. Another model which is studied intensively in recent years, it is the Kuramoto model [8]. In the model the collective behavior of the elements is considered, whose rhythmical activity is described in terms of a physical variable evolving regularly in time. Essentially the behavior of each element is similar to that of an oscillator, but the oscillators belonging to the ensemble are not necessarily identical. They can be characterized for example, by different frequencies. In the frame of this model with mean-field coupling among oscillators, the effect of phase synchronization was also discovered.

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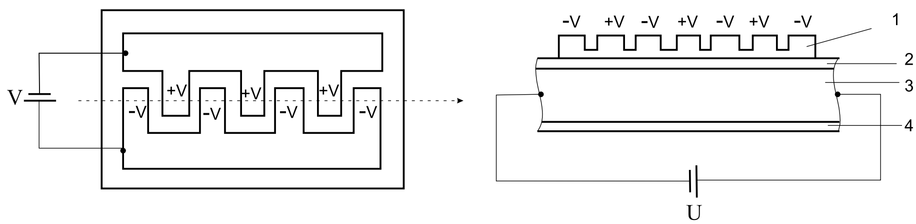

Fig. 1. A sketch of the generic microstructure (micro-undulator): 1 – metal grating, 2 – insulator layer, 3 – semiconductor GaAs, 4 – substrate. Bias V applied to the grating enables to create periodic electric field, bias U applied to the semiconductor determines electron drift velocity v_d.

Our interest in cooperative N^2-effect was inspired mainly by the searching for the new possibilities to generate radiation in THz frequency domain. The last one in its turn is motivated by the growing interest in developing the new sources of THz radiation because of variety of its possible applications ranging from security service [9] to biochemistry and medicine [10]. As it was stressed in [11],

'*The location of the THz field spectrum between the electronic and photonic domains implies that optical or electronic, or even better a mixture of optical and electronic means, can be employed for THz field generation, detection and processing*.'

In the paper [12] we considered the cooperative N^2-effect which appears in the semiconductor structure with grating due to interplay and combining of two effects: undulator-like radiation and Gunn effect. The aim of [12] was to demonstrate that initial correlations in the ensemble of classical oscillators (electrons in Gun-effect diode treated in classical terms) can lead to the cooperative radiation (N^2-effect) in a structure composed of Gunn-effect diode and a properly matched grating, but not to super-radiance. The initial correlations of electrons in such structure occurs simply because the thickness of electron bunch in Gunn effect can be much smaller than the radiation wave length generated due to undulator effect. The result is that due to combining of these two effects, the pulses of radiation occur with the frequencies of about few THz and whose power is proportional to N^2.

Well then, the purpose of this paper is to answer the following question: does the phase synchronization effect and as the consequence, the N^2 effect similar to some extent to the super-radiance, occur in the ensemble of classical nonlinear oscillators with different frequencies and the phases distributed at random? Or in other words, can the effect resembling Dicke super-radiance can develop in the semiconductor structure with grating similar to that considered in [12], which however differs in the respect that there is no Gunn effect and hence, there is lack of initial correlations in the ensemble of classical nonlinear oscillators.

2. The model

Suppose we have a microstructure very similar to that described in [13] or that reported in [14], which was grown on semi-insulating GaAs substrate and with the grating (electrodes arranged in a periodic sequence) on the top (Fig. 1). If the electric bias is applied to the electrodes, then a weak periodic potential modulation arises within the GaAs-sample and this periodic electric field becomes very similar to the periodic electric field used in some types of undulators (in most of the cases in undulators the periodic arrangement of magnets is used, however the periodic electric field is also employed, especially in free electron lasers (FEL)). The quantum-mechanical model of a two-level system describes well the spontaneous emission of atoms and molecules in electron and vibrational transitions, and is widely used in quantum optics. To describe the spontaneous and induced emission in the millimeter and submillimeter bands it becomes necessary to use the model of classical oscillators, since the emitted-photon energy in this range is low and to obtain high-power radiation the energy resource of each oscillator must exceed many times the energy of a single photon.

In addition, for emission of weakly relativistic electrons in a constant field, such as for instance, in cyclotron-resonance masers, the oscillating-electron anharmonicity due to the relativistic correction to the Hamiltonian can be much less than the constant of the interaction with the radiation field. Therefore many levels participate in the emission and absorption processes even in the resonance approximation, and the two-level approximation becomes irrelevant. Instead, the model of nonlinear classical oscillators seems to be most appropriate to describe the situation, that is why in [12] namely this model was used.

The radiation field produced by the charged particles moving in such structure should consist of narrow spectral lines whose frequencies are $\omega = 2\pi kT^{-1}/(1 - L\cos\vartheta/(cT))$, where c is light velocity, k is an integer and ϑ is the observation (excursion) angle. This is exactly the radiation spectrum emitted by the particles in an undulator.

It is clear that in the proposed structure not an ideal narrow spectral line would be observed, but the spectrum of finite width. Since the electrons undergo periodic oscillations in the structure in question, the generated field consists of a wave train with the relative width of about $\Delta\omega/\omega \sim 1/n$ where n is the number of periods of the structure. Other sources of the spectrum broadening are of course the imperfections of grating and the small deviations of electron drift velocities from their average value v_d. It is obvious that the small deviations of spacing between the electrodes as well as velocity deviations are inevitable, and hence, the frequencies of radiation fields generated by the individual electrons will be slightly different. An interesting question is: whether one should take into account the higher harmonics of radiation or it is sufficient to consider only the lowest one? It turns out that if $v \ll c$ (remember, the electron velocity is hardly greater than 10^6 cm s^{-1}), the highest harmonics can be neglected. Another argument in favour of considering only the lowest harmonics is the following. It is well known that if a function of a single variable $f(t)$ is differentiable everywhere within some closed interval and its first derivative is a function of bounded variance, then the coefficients of Fourier series for $f(t)$ decrease as their index k increases, at least as k^{-2} (see [15]). If instead, the smoothness of the function $f(t)$ is of a higher order, then the Fourier coefficients can decrease even faster than that.

Let's consider the system of N classical nonlinear oscillators when each of them is characterized by mass m, charge e, some characteristic length having the physical meaning of the amplitude and which we associate with a period of grating a and the undumped angular frequency ω_i in the presence of an external forcing field with the angular frequency ν. We suppose ν to be equal to $\langle\omega_i\rangle = \omega_0 = a/v_d$, where v_d is the electron drift velocity. We assume also, that ω_i, the frequencies of the individual oscillators are distributed around ω_0 with the deviations not greater than ten per cent.