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Non-zero total correlation means non-zero quantum correlation

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A B S T R A C T

We investigated the super quantum discord based on weak measurements. The super quantum discord is an extension of the standard quantum discord defined by projective measurements and also describes the quantumness of correlations. We provide some equivalent conditions for zero super quantum discord by using quantum discord, classical correlation and mutual information. In particular, we find that the super quantum discord is zero only for product states, which have zero mutual information. This result suggests that non-zero correlations can always be detected using the quantum correlation with weak measurements. As an example, we present the assisted state-discrimination method.

1. Introduction

Quantum measurement plays a key role in quantum mechanics and has certain interesting quantum properties that are rarely seen in everyday life. These properties include the collapse of the wave function, the concept of compatible observables and the contextuality phenomenon. To perform a quantum measurement, one must construct a set of orthogonal projection operators corresponding to the observable eigenvector spaces of a Hermitian operator. The possible outcomes of the measurement correspond to the eigenvalues of the Hermitian operator. This type of measurement is the standard von Neumann measurement, or projective measurement [1]. Recently, the formalism was generalized to the positive-operator-valued measure (POVM) [2], which can capture many phenomena beyond what can be probed by projective measurements.

However, the measurement of a quantum state inevitably disturbs the quantum system, which, in turn, determines what knowledge can be retrieved regarding the measured system. To exert the least influence on the original quantum state, a measurement can be introduced that induces a partial collapse of the quantum state. This is the so-called weak measurement [3–5]. Quantum states can be retrieved with a non-zero success probability when the interaction between the system and the measurement apparatus is weak [6]. It has been shown that any generic measurement can be decomposed into a sequence of weak measurements [7]. Therefore, weak measurements are universal. Furthermore, the reverse process has drawn considerable attention both theoretically and experimentally [8,9] because of its potential applications in quantum information processing [10]. In addition, weak measurements can amplify extremely tiny signals [11,12].

Searching for quantum correlation in a composite system and identifying its role in quantum information processing is one of the fundamental problems in quantum mechanics. Quantum entanglement is widely regarded as having a crucial role in quantum teleportation and superdense coding [2]. Quantum discord [13–15], which is more stringent than quantum entanglement, can effectively elucidate the role of the quantumness of correlations and is different from the classical correlation. Quantum discord has been proven to be present in deterministic quantum computation with one qubit (DQC1) [16] and can be used as a resource in remote state preparation [17]. Furthermore, the consumed discord bounds the quantum advantage in encoded information [18]. Quantum dissonance (or one-side discord) has been proven to be required for optimal assisted discrimination [19–21]. It is known that quantum entanglement can be described and detected using various methods [22–24]. However, quantum discord can exist when entanglement is absent. The quantum discord vanishes for the so-called classical–classical (CC) state, the classical–quantum (CQ) state and the quantum–classical (QC) state [25–27].

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However, studies indicate that the quantum advantage may exist even for vanishing discord [28]. It should, then, be possible to construct a measure of quantum correlation that always exists, except for product states. A good candidate for this measure is the super quantum discord, which is an extension of the quantum discord based on weak measurements [29]. The super quantum discord is always larger than the normal discord induced by strong (projective) measurements, which suggests that the super quantum discord can capture significantly more correlation information. Furthermore, super discord can result in the improvement of the entropic uncertainty relations [30,31]. Thus, we can ask: what is the criterion by which the super quantum discord exists in a quantum system? Can super discord exist in a quantum information model in which quantum discord and entanglement do not exist? In this article, we provide a necessary and sufficient condition for the vanishing of the super discord in terms of classical correlation, mutual information and normal discord. Our results indicate that the quantum correlation measured by the super quantum discord always exists, except when there is no correlation. Thus, we can confirm the expectation that all correlations can be detected from the perspective of quantum correlation. We further illustrate that super discord can occur in optimal assisted state discrimination on both sides, whereby only one-side quantum discord is present and entanglement is unnecessary.

This article is organized as follows. In Section 2, we review the definition and several properties of super discord. In Section 3, we provide a series of necessary and sufficient conditions for the vanishing of the super discord. An illustration of the super discord present on both sides in optimal assisted state discrimination is given in Section 4. Finally, we present our summary in Section 5.

2. The concept and properties of super discord

Consider the bipartite state ρ on the space $\mathcal{H}_A \otimes \mathcal{H}_B$. Let $\{\pi_k\}$ be one-dimensional von Neumann projectors, and the probability is given by $p_k = \mathrm{Tr}(I \otimes \pi_k)\rho(I \otimes \pi_k)$. The completeness of the operators $\{\pi_k\}$ implies the formula $\sum_k p_k = 1$. Next, $S(\rho) := -\mathrm{Tr}\,\rho \log \rho$ is the von Neumann entropy, whereby "log" denotes "$\log_2$" throughout the article. We refer to ρ_A, ρ_B as the reduced density operators of ρ. Then, we denote the mutual quantum information by $I(\rho) := S(\rho_A) + S(\rho_B) - S(\rho)$ and the classical correlation by $C(\rho) := \max_{\pi_k} I(\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k))$ [13,14,32]. Both these terms are non-negative because the mutual information is non-negative [2].

The quantum discord for ρ is defined as the difference between the mutual information and the classical correlation as follows [14,25]:

$$\begin{aligned} D(\rho) &= I(\rho) - C(\rho) \\ &= S(\rho_B) - S(\rho) + \min_{\pi_k} \sum_k p_k S\left(\frac{(I \otimes \pi_k)\rho(I \otimes \pi_k)}{p_k}\right). \end{aligned} \quad (1)$$

It is known that [26,27] the ("right") discord is zero if and only if $\rho = \sum_i p_i \rho_i \otimes |\varphi_i\rangle\langle\varphi_i|$, where the $|\varphi_i\rangle$ are o.n. basis. This criterion defines the so-called classical state of the system B.

Next, we recall the super quantum discord $D_w(\rho)$ for the two-qubit states ρ introduced in [29], which is defined as follows:

$$D_w(\rho) := \min_{\{\pi_0, \pi_1\}} S_w\big(A \bigm| \{P^B(x)\}\big) - S(A|B), \quad (2)$$

where the conditional entropy $S(A|B) = S(\rho) - S(\rho_B)$;

$$S_w\big(A \bigm| \{P^B(x)\}\big) = p(x)S(\rho_{A|P^B(x)}) + p(-x)S(\rho_{A|P^B(-x)}), \quad (3)$$

$$p(\pm x) = \mathrm{Tr}\big(\big(I \otimes P^B(\pm x)\big)\rho\big(I \otimes P^B(\pm x)\big)\big), \quad (4)$$

$$\rho_{A|P^B(\pm x)} = \frac{1}{p(\pm x)} \mathrm{Tr}_B\big(\big(I \otimes P^B(\pm x)\big)\rho\big(I \otimes P^B(\pm x)\big)\big), \quad (5)$$

$$P(x) = \sqrt{\frac{1-\tanh x}{2}}\pi_0 + \sqrt{\frac{1+\tanh x}{2}}\pi_1, \quad (6)$$

$$P(-x) = \sqrt{\frac{1+\tanh x}{2}}\pi_0 + \sqrt{\frac{1-\tanh x}{2}}\pi_1, \quad (7)$$

and $x \in R \setminus \{0\}$ is a parameter that describes the strength of the measurement process. Using Eq. (2), we can write $D_w(U \otimes V\rho U^\dagger \otimes V^\dagger) \leqslant D_w(\rho)$ with arbitrary unitary U, V. One can similarly obtain $D_w(U \otimes V\rho U^\dagger \otimes V^\dagger) \geqslant D_w(\rho)$; thus, we can write

$$D_w\big(U \otimes V\rho U^\dagger \otimes V^\dagger\big) = D_w(\rho). \quad (8)$$

In other words, the super discord is invariant up to the local unitary. This property is the same as that of the normal discord.

Using Eqs. (6) and (7), we can obtain the completeness relation as follows:

$$\pi_0 + \pi_1 = f\, P(x)^\dagger P(x) + P(-x)^\dagger P(-x) = I. \quad (9)$$

Using Eqs. (4) and (9), we observe that the probability sum is equal to one as follows:

$$p(x) + p(-x) = 1. \quad (10)$$

Using the concavity of the von Neumann entropy and Eqs. (3) and (5), we can easily obtain $I(\rho) \geqslant D_w(\rho)$. By incorporating the theorem of [29], we can obtain

$$I(\rho) \geqslant D_w(\rho) \geqslant D(\rho) \quad (11)$$

for any two-qubit states. However, these three quantities are not quantitatively related to the classical correlation. It follows from Ref. [33] that the difference $C(\rho) - D(\rho)$ can be either positive or negative for the two-qubit Bell diagonal states ρ in [33]; see [34]. Nevertheless, we derive the relations among the classical correlation, mutual information, super discord and discord for product states in the next section.

3. Condition for zero super discord

Similar to the case of discord, we can ask the following question: what are the states ρ whose super discord is zero? According to Eq. (11) and [26,27], such states ρ must be classical in the system B. However, the converse is not manifestly true; see Theorem 1 below. Thus, we require a preliminary lemma. It is known that the classical correlation is zero for a product state [13]. We can show that the inverse is also true.

Lemma 1. *Any bipartite state ρ that satisfies $C(\rho) = 0$ is a product state, i.e., $\rho = \rho_A \otimes \rho_B$.*

Proof. By definition, the condition $C(\rho) = 0$ implies that $I(\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k)) = 0$ holds for any $\{\pi_k\}$. By the subadditivity of the von Neumann entropy, the state $\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k)$ is a product state. By tracing out the system A or B, we obtain

$$\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k) = \rho_A \otimes \sum_k \pi_k \rho_B \pi_k \quad (12)$$

for any $\{\pi_k\}$. Let $\rho_B = \sum_i p_i |b_i\rangle\langle b_i|$ be the spectral decomposition, and we can assume $\rho = \sum_{ij} \rho_{ij} \otimes |b_i\rangle\langle b_j|$. By choosing $\pi_i = |b_i\rangle\langle b_i|$ in Eq. (12), we obtain $\rho_{ii} = p_i \rho_A$, $\forall i$. Using the normalization condition $\sum_i p_i = 1$, we obtain $\rho = \rho_A \otimes \rho_B + \sum_{i \neq j} \rho_{ij} \otimes |b_i\rangle\langle b_j|$. By substituting this expression for ρ into Eq. (12), we obtain

$$\sum_k (I \otimes \pi_k)\left(\sum_{i \neq j} \rho_{ij} \otimes |b_i\rangle\langle b_j|\right)(I \otimes \pi_k) = 0 \quad (13)$$