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Cumulative quantum work-deficit versus entanglement in the dynamics of an infinite spin chain

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ABSTRACT

We find that the dynamical phase transition (DPT) in nearest-neighbor bipartite entanglement of time-evolved states of the anisotropic infinite quantum XY spin chain, in a transverse time-dependent magnetic field, can be quantitatively characterized by the dynamics of an information-theoretic quantum correlation measure, namely, quantum work-deficit (QWD). We show that only those nonequilibrium states exhibit entanglement resurrection after death, on changing the field parameter during the DPT, for which the cumulative bipartite QWD is above a threshold. The results point to an interesting inter-relation between two quantum correlation measures that are conceptualized from different perspectives.

1. Introduction

Correlations form important indicators of the physics of a system involving a large number of interacting particles. The behavior of quantum correlations [1,2] in macroscopic phases of such many-body systems is potentially important in understanding nonclassical properties such as critical phenomena, and quantum fluctuations [3,4]. In particular, entanglement [1] has been widely used in many-body physics to study near-critical behavior, phase transitions, and the general evolution in spin systems [4].

In recent years, the concept of nonclassical correlations in quantum systems has been taken beyond quantum entanglement, due to the existence of interesting few-body quantum phenomena where entanglement is absent [5]. This has led to the formulation of information-theoretic quantifiers of quantum correlation, independent of the entanglement-separability criteria, based on the thermodynamics of local and global measurement strategies [6,7]. These measures have been used to study various aspects of quantum information, and in particular, have been applied to investigate many-body phenomena such as quantum phase transitions [8], correlation dynamics in many-body systems [9] (for a review, see [2]) and in open quantum systems [10]. An important example of such an information-theoretic measure of nonclassical correlations is the quantum work deficit [7].

In this article, we study the behavior of quantum work deficit (QWD) [7] for an infinite anisotropic quantum XY spin chain in a transverse time-dependent magnetic field. QWD is the difference in negentropy ("work") that can be extracted by using global and local heat engines [7,11]. The concept of QWD is based on the fact that information can be treated as a thermodynamic resource [12] and it is hence defined as the difference between the amount of pure states that can be extracted under global operations and that under certain local operations. The difference accounts for the missing information ("resource") that can be attributed to the presence of quantum correlations in the system, independent of entanglement.

The nearest-neighbor entanglement of the nonequilibrium time-evolved state of an anisotropic infinite XY spin chain in a transverse time-dependent field is known to exhibit a dynamical phase transition (DPT) [13] for a fixed, short time. Specifically, entanglement dies for a small initial magnetic field while the state becomes entangled with the increase of magnetic field. We find that the nature of the DPT of bipartite entanglement can be characterized by studying the dynamics of QWD. In particular, we show that entanglement death and possible resurrection with changing initial magnetic field as observed during the DPT, can be inferred by the "cumulative" QWD in the system during the evolution. Only those nonequilibrium states exhibit revival after death for which the cumulative QWD is above a threshold value. The results are independent of the anisotropy in the quantum XY model.

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The quantitative relation exhibited between QWD and entanglement is potentially an interesting observation. Although for general mixed quantum states, there may not exist any direct relation between information-theoretic and entanglement measures (cf. [14]), in these specific class of systems, we establish a quantitative relation between them. The results may be of interest in investigating the interplay of such quantum correlation measures in the dynamics of generic quantum many-body systems.

2. The XY spin chain

The Hamiltonian for the quantum spin chain that we consider is given by $H(t) = H_{int} - h(t)H_{mag}$, where the external magnetic field has the form $H_{mag} = \sum_i S_i^z$. The interaction term is given by $H_{int} = J\sum_i(\mathcal{A}S_i^x S_{i+1}^x + \mathcal{B}S_i^y S_{i+1}^y)$, where J measures the interaction strength, and $S^j = \frac{1}{2}\sigma^j$ $(j = x, y, z)$ are one-half of the Pauli spin matrices at the corresponding site. The spin coupling constants can be defined in terms of the anisotropy parameter γ as $\mathcal{A} = 1+\gamma$ and $\mathcal{B} = 1-\gamma$, where $\gamma \neq 0$. The anisotropy parameter is chosen to ensure $[H_{int}, H_{mag}] \neq 0$, and hence the external field evokes a non-trivial response in the system dynamics. The external transverse field is applied in the form of a finite initial perturbation at $t = 0$: $h(0) = a > 0$, for $t = 0$. The external field vanishes at $t > 0$: $h(t) = 0$, for $t > 0$. Hence, the dynamics of the system at any time t depends on the initial field a. Note that J and a have the units of energy, while γ is dimensionless.

To investigate the nearest neighbor (NN) correlation properties of the anisotropic XY spin chain, we require the two-site reduced density matrix of the nonequilibrium time-evolved state. If we consider the initial state of the system as a canonical equilibrium state at temperature T, the nonequilibrium two-site reduced density matrix, ρ_{12}^β, at time t is given by [15]

$$\rho_{12}^\beta(t) = \frac{1}{4}\Big[I\otimes I + M^z(t)\big(\sigma^z\otimes I + I\otimes\sigma^z\big) + l^{xy}(t)\big(\sigma^x\otimes\sigma^y + \sigma^y\otimes\sigma^x\big) + \sum_{j=x,y,z} l^{jj}(t)\sigma^j\otimes\sigma^j\Big], \tag{1}$$

where $l^{ij}(t)$ $(i, j = x, y, z)$ are the NN classical correlation functions and M^z is the transverse magnetization. Upon diagonalizing the Hamiltonian, using the Jordan–Wigner and Fourier transformations, the NN classical correlation functions and the magnetization in Eq. (1) are given as follows [15]:

$$l^{xy}(t) = l^{yx}(t) = s(t),$$
$$l^{xx}(t) = g(-1, t), \qquad l^{yy}(t) = g(1, t), \tag{2}$$

where $g(i', t)$ (for $i' = \pm 1$) and $s(t)$, for initial temperature $T = 0$, are given by

$$g(i', t) = \frac{\gamma}{\pi}\int_0^\pi d\phi\, \frac{\sin(i'\phi)\sin\phi}{\Lambda(\tilde a)\Lambda^2(0)} \times \big\{\gamma^2\sin^2\phi + (\cos\phi - \tilde a)\cos\phi + \tilde a\cos\phi\cos\big[2\Lambda(0)\tilde t\big]\big\} - \frac{1}{\pi}\int_0^\pi d\phi\, \frac{\cos\phi}{\Lambda(\tilde a)\Lambda^2(0)} \times \big(\big\{\gamma^2\sin^2\phi + (\cos\phi - \tilde a)\cos\phi\big\} \times \cos\phi - \tilde a\gamma^2\sin^2\phi\cos\big[2\Lambda(0)\tilde t\big]\big), \tag{3}$$

$$s(t) = -\frac{\gamma\tilde a}{\pi}\int_0^\pi d\phi\, \sin^2\phi\, \frac{\sin[2\tilde t\Lambda(0)]}{\Lambda(\tilde a)\Lambda(0)}. \tag{4}$$

The magnetization is given by

$$M^z(t) = \frac{1}{\pi}\int_0^\pi d\phi\, \frac{1}{\Lambda(\tilde a)\Lambda^2(0)}\big\{\cos\big[2\Lambda(0)\tilde t\big]\gamma^2\tilde a\sin^2\phi - \cos\phi\big[(\cos\phi - \tilde a)\cos\phi + \gamma^2\sin^2\phi\big]\big\}. \tag{5}$$

Here, $\Lambda(x) = \{\gamma^2\sin^2\phi + [x - \cos\phi]^2\}^{\frac{1}{2}}$, and $\tilde a = a/J$, $\tilde t = Jt/\hbar$. In our analysis, the dimensionless variables $\tilde a$ and $\tilde t$ are used as the initial field and time parameters, respectively.

3. Measures of quantum correlation

Entanglement (logarithmic negativity). For analyzing the properties of bipartite entanglement of the nonequilibrium time-evolved state of the anisotropic XY spin chain, the logarithmic negativity (LN) proves to be a useful computational measure of entanglement [16]. The definition of logarithmic negativity is based on the Peres–Horodecki separability criterion [17,18]. The negativity of the partial transpose of any two-party state is a sufficient condition for bipartite entanglement. The condition is necessary and sufficient for two qubits [18]. The LN of an arbitrary bipartite state, ρ_{12}, is defined as

$$E_{\mathcal{N}}(\rho_{12}) = \log_2\big\|\rho_{12}^{T_A}\big\|_1 \equiv \log_2\big[2\mathcal{N}(\rho_{12}) + 1\big], \tag{6}$$

where $\mathcal{N}(\rho_{12}) = (1/2)(\|\rho_{12}^{T_1}\|_1 - 1)$ is called the "negativity", and $\|\rho_{12}^{T_1}\|_1$ is the trace norm of the partially transposed state, $\rho_{12}^{T_1}$, of ρ_{12}. The negativity, $\mathcal{N}(\rho_{12})$, is thus the sum of the absolute values of the negative eigenvalues of $\rho_{12}^{T_1}$. For general mixed states, the LN is an upper bound for distillable entanglement [16].

Quantum work-deficit. Quantum work deficit (QWD) is based on the concept that information is a thermodynamic resource [12] and its utility and dynamics is governed by similar laws. It is defined as the difference between the amount of extractable pure states under suitably restricted global and local operations [7]. Hence, QWD is an information-theoretic measure of quantum correlation, independent of the entanglement-separability criteria.

The allowed class of global operations on a quantum state ρ_{12} is called "closed operations" (CO). This set of operations consists of: (a) unitary operations, and (b) dephasing the quantum state ρ_{12} using an orthonormal projector set $\{X_i\}$ such that $\rho_{12} \to \sum_i X_i\rho_{12}X_i$, where the operations are defined on the Hilbert space $(\mathcal{H})$ of ρ_{12}. The amount of pure states extractable under CO can be shown to be $I_{\mathrm{CO}} = N - S(\rho_{12})$. Here, $N = \log_2\dim(\mathcal{H})$ and $S(\rho)$ denotes the von Neumann entropy of the state ρ. For the case of local operations, the allowed class of operations is closed local operations and classical communication (CLOCC) which consists of: (a) local unitary operations (b) dephasing locally and communicating a dephased subsystem to the other party over a noiseless quantum channel. The amount of pure states extractable under CLOCC is given by $I_{\mathrm{CLOCC}} = N - \inf_{\Lambda\in\mathrm{CLOCC}}[S(\rho_1') + S(\rho_2')]$, where $\rho_i' = \mathrm{Tr}_j(\Lambda(\rho_{ij}))$ $(i, j = 1, 2; i \neq j)$. The work deficit $(\Delta(\rho_{12}))$ is then given by

$$\Delta(\rho_{12}) = I_{\mathrm{CO}}(\rho_{12}) - I_{\mathrm{CLOCC}}(\rho_{12}). \tag{7}$$

The QWD can also be defined with respect to two-way communication of the dephased subsystem, although, it is hard to compute for arbitrary quantum states. Hence, we restrict ourselves to the case where one-way communication is permitted. For two-qubit systems, the dephasing involves a projection-valued