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Dimerization, trimerization and quantum pumping

Huaiming Guo

Department of Physics, Beihang University, Beijing, 100191, China

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ABSTRACT

We study one-dimensional topological models with dimerization and trimerization and show that these models can be generated using interaction or optical superlattice. The topological properties of these models are demonstrated by the appearance of edge states and the mechanism of dimerization and trimerization is analyzed. Then we show that a quantum pumping process can be constructed based on each one-dimensional topological model. The quantum pumping process is explicitly demonstrated by the instantaneous energy spectrum and local current. The result shows that the pumping is assisted by the gapless states connecting the bands and one charge is pumped during a cycle, which also defines a nonzero Chern number. Our study systematically shows the connection of one-dimensional topological models and quantum pumping, and is useful for the experimental studies on topological phases in optical lattices and photonic quasicrystals.

1. Introduction

The discovery of topological insulators has aroused the interests in the study of topological phase of matter [1–4]. Specifically recently a lot of works focus on one dimension (1D) [5–16]. On the one hand the physics of 1D is as rich as higher dimensions, but easier to understand. On the other hand many 1D systems can be realized experimentally in ultracold atomic system [13–15] and photonic quasicrystals [12], thus provide platforms to test the theoretical predictions.

In 1979, Su et al. suggest a model to study the solitons in polyacetylene [17], which has become a famous 1D topological model and many studies have been carried on it. Later Su and Schrieffer also study a 1D one-third filled Peierls system and find the fractional topological excitation associated with the kinks [18]. The above two models are also known as dimerized and trimerized models. The physical properties of the two models represent a new kind of order called topological order [19], which is beyond Landau symmetry breaking theory. Generally the topological order contains two different types: the intrinsic and symmetry protected ones. In 1D, only symmetry protected topological order exists.

Recently there appear works studying the topological phases in 1D optical superlattices, including topological phase transition, fractional topological phases, topological Mott insulators, and even topological superconductor [5,8,11]. The based model of the above studies is simple, but the phenomena it exhibits are rich. More inspiringly, this model has been realized using photonic quasicrystals [12] and is very possible to be realized in the optical lattices. These theoretical and experimental achievements motivate us to have more understanding on this 1D model and related physics.

In this paper, we show that the topological phase induced by optical superlattice is closely related to the trimerized model, and besides the interaction can generate topological phase as that in the dimerized model. We also connect the 1D topological phase with the quantum pumping and the Chern number. These results provide more understanding on the 1D topological phase and are useful for the experimental studies on topological phases in optical lattices and photonic quasicrystals. The paper is organized as follows. In Section 2, the dimerized and trimerized models are revisited and the topological properties are demonstrated with edge states on open chains. In Section 3, we generate models exhibiting similar properties as the dimerized and trimerized models using the interaction and optical superlattice and discuss the reason for that. In Section 4, we show that based on each 1D topological model, a quantum pumping process can be constructed, which is explicit shown by the instantaneous energy spectrum and local current, and in the process a nonzero Chern number is defined. Finally we conclude the work in Section 5.

2. The models

The first model is with the dimerization, which writes [17],

$$H_1 = \sum_i (t_0 + \delta t) c_{Ai}^\dagger c_{Bi} + (t_0 - \delta t) c_{Ai+1}^\dagger c_{Bi} + h.c., \tag{1}$$

where $c_i^\dagger (c_i)$ is the creation (annihilation) operator of the fermion, and A, B represent two different sites of a unit-cell. The hopping

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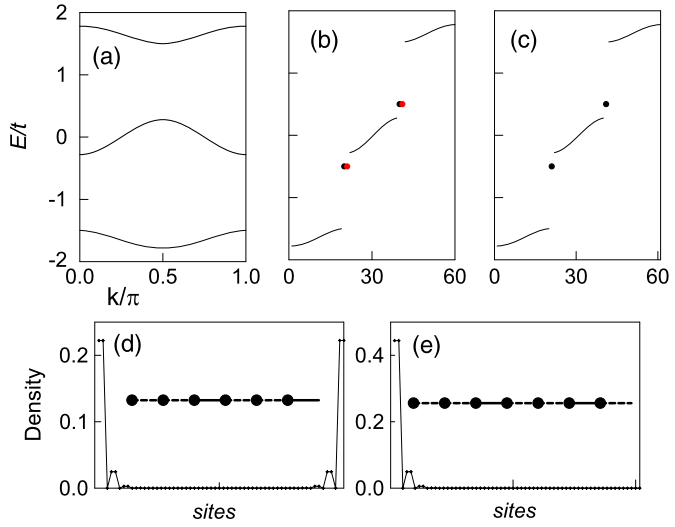

Fig. 1. (Color online.) The energy spectrum of Hamiltonian equation (2) with $\delta t = 0.5$: (a) periodic boundary condition with $N = 3N_u$; (b) and (c) open boundary condition with $N = 3N_u$ and $N = 3N_u + 1$ respectively. Here $N_u = 20$ is the number of unit-cells. (d) and (e) the distribution of the lower in-gap states in (b), (c) respectively. In (b) the zero mode is two-fold degenerate, and one is with the red color for clarity.

amplitude t_0 is set to be the unit of the energy ($t_0 = 1$). In the reciprocal space, the Hamiltonian is written as $H_1 = \sum_k \psi_k^\dagger H_1(k)\psi_k$ with $\psi_k = (c_{Ak}, c_{Bk})^T$ and

$$H_1(k) = 2t_0 \cos k\sigma_x - 2\delta t \sin k\sigma_y,$$

with $\sigma_{x,y}$ the Pauli matrices. The energy spectrum is given by

$$E_{\mathbf{k}} = \pm\sqrt{4t_0^2 \cos^2 k + 4\delta t^2 \sin^2 k},$$

with the gap $4|\delta t|$ at the Dirac point $k = \pi/2$ (or $3\pi/2$). There are two topologically distinct phases depending on the sign of δt: for $\delta t > 0$ it is topological trivial; for $\delta t < 0$ the system shows topological property with zero-energy states localized near the ends. The topological invariant characterizing the two phases is a Berry phase which is zero for $\delta t > 0$ and π for $\delta t < 0$.

The second model we consider is with the trimerization [18],

$$H_2 = \sum_i t_1 c_{Ai}^\dagger c_{Bi} + t_2 c_{Bi}^\dagger c_{Ci} + t_3 c_{Ci}^\dagger c_{Ai+1} + h.c. \tag{2}$$

In this model, each unit-cell consists of three sites, A, B and C. There are three configurations with perfect trimerization: 1, $t_1 = t_2 = t_0 - \delta t$ and $t_3 = t_0 + \delta t$; 2, $t_2 = t_3 = t_0 - \delta t$ and $t_1 = t_0 + \delta t$; 3, $t_1 = t_3 = t_0 - \delta t$ and $t_2 = t_0 + \delta t$. In the following we discuss the first case and the others are similar. In the momentum space, the Hamiltonian writes as $H_2 = \sum_k \psi_k^\dagger H_2(k)\psi_k$ with $\psi_k = (c_{Ak}, c_{Bk}, c_{Ck})^T$ and

$$H_2(k) = \begin{pmatrix} 0 & t_0 - \delta t & (t_0 + \delta t)e^{-ik} \\ t_0 - \delta t & 0 & t_0 - \delta t \\ (t_0 + \delta t)e^{ik} & t_0 - \delta t & 0 \end{pmatrix}$$

For $\delta t \neq 0$ the system is gapped at $\frac{1}{3}$ and $\frac{2}{3}$ filling and both the gaps are $\frac{3(t_0+\delta t)}{2} - \frac{1}{2}\sqrt{9t_0^2 - 14t\delta t + 9\delta t^2}$. For $\frac{1}{3}$ filling it is located at $k = 0$ while at $k = \pi$ for $\frac{2}{3}$ filling. Depending on the sign of δt and the value of the chain length $N \bmod 3$, there appear different edge states. When $\mathrm{mod}(N, 3) = 0$, there appear edge states at both ends for $\delta t > 0$ but none for $\delta t < 0$. When $\mathrm{mod}(N, 3) = 1$, there appear one edge state at the left end for $\delta t > 0$ but none for $\delta t < 0$. When $\mathrm{mod}(N, 3) = 2$, there appear one edge state at the left end for $\delta t > 0$ but at the right end for $\delta t < 0$ (Fig. 1).

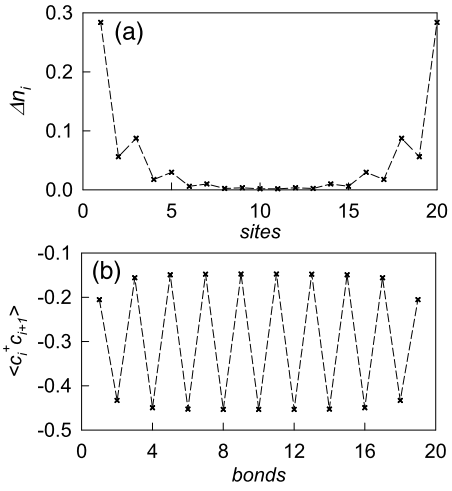

Fig. 2. (a) The distribution of the quasiparticles added or removed from the half-filling system with OBC. (b) the average value of the hopping amplitude of each bond on an open chain. The calculations are for model equation (3) with $V_1 = 1$ and the length of the chain $N = 20$.

3. Generating δt from interaction or optical superlattice

In the previous section, we revisit the dimerized and trimerized models. Next we drop δt terms in both models and dynamically generate δt from interaction or optical superlattice.

We firstly generate a model showing similar properties to Eq. (1) from nearest neighbor (NN) interaction. The Hamiltonian under consideration is:

$$H_1' = t_0 \sum_i (c_i^\dagger c_{i+1} + h.c.) + \sum_{\langle ij \rangle} V_{ij} n_i n_j, \tag{3}$$

where the NN interaction is with alternating strength, i.e., $V_{ij} = (-1)^i V_1$ supposing $i < j$. For $V_1 < 0$ the system is trivial, while for $V_1 > 0$ the system becomes nontrivial when the system shows similar properties as Eq. (1) at $\delta t < 0$. For a many-body system, the topological properties can manifest itself from the edge states of quasiparticles. The energy of the quasiparticle added to a system with n electrons can be defined as $\Delta E_n = E_{n+1}^0 - E_n^0$, where E_n^0 is the ground energy of a system with n particles [7]. For the case of $V_1 > 0$ there appear states in the gap of the quasiparticle energy spectrum as the boundary condition changes from periodic (PBC) to open (OBC) ones. The distribution of the in-gap state can be calculated which is defined as: $\Delta n_i = \langle \psi_{n+1}^0 | \hat{n}_i | \psi_{n+1}^0 \rangle - \langle \psi_n^0 | \hat{n}_i | \psi_n^0 \rangle$, where $\hat{n}_i = c_i^\dagger c_i$ is the electron number operator on site i and ψ_n^0 is the ground-state wave function of the system with n electrons [7]. The distribution of the in-gap state is shown in Fig. 2, which mainly distributes near the edges. So for the case of $V_1 > 0$ the system exhibits nontrivial topological properties.

We further calculate the average value of the hopping amplitude of each bond, which provides an explanation of the nontrivial phase in Eq. (3). As shown in Fig. 2, due to the existence of the alternating NN interactions, the effective hopping amplitude $\langle c_i^\dagger c_{i+1} \rangle$ becomes alternating, which let Eq. (3) show similar properties as the dimerized model equation (1).

Next we effectively generate Hamiltonian equation (2) from a model with a uniform nearest neighbor hopping and an optical superlattice, which writes,

$$H_2' = t_0 \sum_i (c_i^\dagger c_{i+1} + h.c.) + \sum_{i=1}^N V_i n_i, \tag{4}$$

with $V_i = V_2 \cos(\frac{2\pi}{3} i + \frac{2\pi}{3})$. With this superlattice a unit-cell still consists of three sites A, B, C, and the on-site potentials are