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Delay-range-dependent stability for fuzzy BAM neural networks with time-varying delays

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ABSTRACT

This Letter considers the problem of delay-range-dependent stability for fuzzy bi-directional associative memory (BAM) neural networks with time-varying interval delays. Based on Lyapunov–Krasovskii theory, the delay-range-dependent stability criteria are derived in terms of linear matrix inequalities (LMIs). By constructing new Lyapunov–Krasovskii functional, stability conditions are dependent on the upper and lower bounds of the delays, which is made possible by using some advanced techniques for achieving delay dependence. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

It is well known that the dynamics of neural networks such as cellular neural networks (CNNs) [1], Hopfield neural networks (HNNs) [2] and bidirectional associative memory (BAM) [3] have been deeply investigated in recent years due to its applicability in solving some image processing, signal processing, optimization and pattern recognition problems. Therefore, the investigation on the dynamical behavior of the neural networks is very important and significant [4–10]. The bidirectional associative memory (BAM) model known as an extension of the unidirectional autoassociator of Hopfield. This kind of neural networks has been widely studied due to its promising potential for applications in different fields such as combinatorial optimization, pattern recognition, signal and image process, etc. Thus, the stability analysis is a important step for the design and applications for this neural network. The stability analysis of BAM neural networks with delays has attracted considerable interest, see, for example [11–19] and references therein. In electronic implementation of analog neural networks, nevertheless, the delays are usually time-varying due to the finite switching speed of amplifiers. It has been showed that time delays are often a source of instability for neural networks [20]. Therefore, more and more results have been reported for delayed BAM neural networks [17,18,21].

Among various methods developed for the analysis and synthesis of complex nonlinear systems, fuzzy logic control is an attractive and effective rule-based one. In many of the model-based fuzzy control approaches, the well-known Takagi–Sugeno (T–S) fuzzy model [22] is a popular and convenient tool in functional approximations. During the last decade, the stability analysis and controller synthesis problem for systems in T–S fuzzy model has been studied extensively and numerous methods have been proposed in [23–25]. However, in contrast to the pure neural network or fuzzy system, the fuzzy neural network possesses both their advantages [26,27]. It combines the capability of fuzzy reasoning in handling uncertain information [28] and the capability of artificial neural networks in learning from process. It has been showed that fuzzy neural network can approximate a wide range of nonlinear functions to any desired degree of accuracy under

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certain condition [29]. In recent years, the concept of incorporating fuzzy logic into a neural network has grown into a popular research topic [30–36]. In [30], the global asymptotic stability problem of T–S fuzzy BAM neural networks with time-varying delays and parameter uncertainties is considered. In [30], the generalized T–S fuzzy models can be used to represent some complex nonlinear systems by having a set of nonlinear delayed BAM neural networks as its consequent parts [23]. However, the results in [30] are delay-independent. To the best of our knowledge, few results on delay-range-dependent stability have been reported for T–S fuzzy BAM neural networks with time-varying interval delays.

In this Letter, we will give the delay-range-dependent sufficient condition which guarantees the uniqueness and global stability of the equilibrium solution for fuzzy BAM neural networks with time-varying interval delays. By using free-weighting matrix method [37,38], the stability criteria for the fuzzy delayed BAM neural networks are expressed in the form of LMIs. A numerical example will be provided to illustrate the usefulness and less conservativeness of the developed techniques.

Notation. The notations in this Letter are quite standard. The superscript "T" stands for the transpose of a matrix; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X > Y$ ($X \geqslant Y$) means that the $X - Y$ is positive definite (positive semi-definite), respectively; I is the identity matrix with appropriate dimensions; The symmetric terms in a symmetric matrix are denoted by "$*$"; Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

The model of BAM neural networks with time-varying delays can be expressed as follows:

$$\begin{cases} \dot u_i(t) = -a_i u_i(t) + \sum_{j=1}^{n} b_{ji} F_j(v_j(t)) + \sum_{j=1}^{n} c_{ji} F_j(v_j(t-\tau(t))) + I_i, \\ \dot v_j(t) = -d_j v_j(t) + \sum_{i=1}^{m} e_{ij} G_i(u_i(t)) + \sum_{i=1}^{m} f_{ij} G_i(u_i(t-d(t))) + \bar I_j, \end{cases} \tag{1}$$

for $i = \{1, 2, \dots, m\}$, $j = \{1, 2, \dots, n\}$, $t > 0$, where $u_i(t)$ and $v_j(t)$ denote the activations of the ith neurons and jth neurons, respectively; $F_j(\cdot)$ and $G_i(\cdot)$ stand for the signal functions of the ith neurons and jth neurons, respectively; a_i and d_j are positive constants, they stand for the rate with which the cell i and j reset their potential to the resting state when isolated from the other cells and inputs: b_{ji}, c_{ji}, e_{ij} and f_{ij} denote the synaptic connection weights; I_i and $\bar I_j$ denote the external inputs at time t. The bounded function $\tau(t)$ and $d(t)$ represent unknown delays of systems and satisfy

$$\tau_m \leqslant \tau(t) \leqslant \tau_M, \qquad \dot\tau(t) \leqslant \tau_\mu, \tag{2}$$

$$d_m \leqslant d(t) \leqslant d_M, \qquad \dot d(t) \leqslant d_\mu. \tag{3}$$

(A) We assume that there exist positive w_i^1, w_j^2 such that

$$\left|F_j(x_1) - F_j(x_2)\right| \leqslant w_j^2 |x_1 - x_2|, \qquad \left|G_i(x_1) - G_i(x_2)\right| \leqslant w_i^1 |x_1 - x_2| \tag{4}$$

for all $x_1, x_2 \in \mathbb{R}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$.

The system (1) is supplemented with initial values given by

$$\begin{cases} u_i(t) = \phi_{ui}(t), \quad t \in [-d_M, 0], \ i = 1, 2, \dots, m, \\ v_j(t) = \phi_{vj}(t), \quad t \in [-\tau_M, 0], \ j = 1, 2, \dots, n, \end{cases}$$

where $\phi_{ui}(t), \phi_{vi}(t)$ are continuous functions defined on $[-d_M, 0]$ and $[-\tau_M, 0]$, respectively.

The system (1) is equivalent to the vector form as follows:

$$\begin{cases} \dot u(t) = -Au(t) + BF(v(t)) + CF(v(t-\tau(t))) + I, \\ \dot v(t) = -Dv(t) + EG(u(t)) + FG(u(t-d(t))) + \bar I, \end{cases} \tag{5}$$

where

$$u = (u_1, u_2, \dots, u_m)^T, \qquad v = (v_1, v_2, \dots, v_n)^T,$$

$$A = \operatorname{diag}(a_1, a_2, \dots, a_m), \qquad D = \operatorname{diag}(d_1, d_2, \dots, d_n), \qquad B = \left[(b_{ji})_{n\times m}\right]^T, \qquad C = \left[(c_{ji})_{n\times m}\right]^T, \qquad I = (I_1, I_2, \dots, I_m)^T,$$

$$E = \left[(e_{ij})_{m\times n}\right]^T, \qquad F = \left[(f_{ij})_{m\times n}\right]^T, \qquad \bar I = (\bar I_1, \bar I_2, \dots, \bar I_n)^T,$$

and nonlinear active functions

$$F\big(v(t)\big) = F_j\big(v_j(t)\big)_{n\times 1}, \qquad F\big(v(t-\tau(t))\big) = F_j\big(v_j\big(t-\tau(t)\big)\big)_{n\times 1},$$

$$G\big(u(t)\big) = G_i\big(u_i(t)\big)_{m\times 1}, \qquad G\big(u(t-d(t))\big) = G_i\big(u_i\big(t-d(t)\big)\big)_{m\times 1}.$$

As above mentioned, it is reasonable to assume that the neural network (5) has only one equilibrium point $u^* = (u_1^*, u_2^*, \dots, u_m^*)$, $v^* = (v_1^*, v_2^*, \dots, v_n^*)$. Then, we will shift the equilibrium points u^* and v^* to the origin. The transformation $x(\cdot) = u(\cdot) - u^*$ and $y(\cdot) = v(\cdot) - v^*$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, put system (5) into the following form:

$$\begin{cases} \dot x(t) = -Ax(t) + Bf(y(t)) + Cf(y(t-\tau(t))), \\ \dot y(t) = -Dy(t) + Eg(x(t)) + Fg(x(t-d(t))), \end{cases} \tag{6}$$

where $g(x(t)) = G(x(t) + u^*) - G(u^*)$, $f(y(t)) = F(y(t) + v^*) - F(v^*)$. Then from (A), we have

$$g^T\big(x(t)\big)g\big(x(t)\big) \leqslant x^T(t) W_1^T W_1 x(t), \qquad f^T\big(y(t)\big)f\big(y(t)\big) \leqslant y^T(t) W_2^T W_2 y(t),$$

where $W_1 = \operatorname{diag}(w_1^1, w_2^1, \dots, w_m^1)$, $W_2 = \operatorname{diag}(w_1^2, w_2^2, \dots, w_n^2)$.