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Acceleration and ejection of interacting ring vortices by radial flow

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ABSTRACT

Exact solution of two-dimensional hydrodynamic equations for symmetrical configuration of four point vortices in the presence of radial flow is found. This solution describes the dynamics of a dipole toroidal vortex (consisting of two counter-rotating vortex rings) in such a flow. It is shown that in a convergent flow the ring vortices are compressed and ejected with acceleration along the symmetry axes of the system. Possible application to the problem of jets formation in active galaxy nuclei is considered.

1. Introduction

It is interesting to consider a movement of a dipole toroidal vortex in a radial flow in connection with a number of possible astrophysical and geophysical applications. One of them — the application to active galaxy nuclei (AGN) in quasars, radio galaxies, Seyfert galaxies — is based on the direct observation of obscuring torus [1] and on conception of toroidal vortices [2] surrounding the AGNs (the external one is observed as obscuring torus). Vortex motion can arise in the tori[1] owing to twisting by wind and radiation. The motion has dipole character by virtue of symmetry of the flow (see Fig. 1 in [2], the figure plane corresponds the x–y plane in this consideration). In the simplest case this motion can be represented as the motion of two counter-rotating toroidal vortices in the radial flow.

In this work the dipole toroidal vortex is regarded as the system of two counter-rotating vortex rings. In the absence of a flow this problem resembles the classical Helmholtz problem [3] about interaction of toroidal vortex with the wall parallel to the plane in which the vortex lies. The wall can be replaced with a mirror image of the vortex and the problem is reduced to interaction of two counter-rotating vortex rings. However, the direction of rotation in our case is opposite to the direction corresponding to the movement of the vortex to the wall which was considered by Helmholtz. (Our version would correspond to the vortex movement away from the wall.)

The dynamics of a vortex ring can be described as the motion of the pair of flat point vortices arising in the cross-section of the ring by symmetry plane [4]. In our case this is a symmetrical system of four vortices (or two vortex pairs) in the radial flow. It admits the Hamiltonian formulation [5] and exact analytical solution of the dynamic problem, which is derived in the following chapters. Let us note that in the absence of the flow the similar problem for symmetrical system of four vortices has been solved by Grobli [6] (see also [7]).

As is known from observations, the bursts which are observed in AGN are accompanied by ejection of separate jet components. (Some of them perform relativistic motion in the form of so-called superluminal sources.) In microquasars, which are the stellar analog of AGN, non-relativistic jet components along with superluminal ones are also observed [8]. Only far enough from the AGN centre the separate components is likely to merge into a continuous jet.

In our model the ejections in AGN are formed due to kinematics of vortex interaction. In this process the radial flow plays essential role affecting the ejection velocity. Although it seems quite para-

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[1] We have to point out that the presence of vortex motion, which plays essential role in the considered model, enables to explain the existence of a thick but cold tori which were observed in [1].

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doxical at the first glance, greater velocities of the ejected components are achieved in the convergent flow. It does not imply the presence of any magnetic field (see the discussion in the end of the Letter).

2. Plane vortices in the radial flow

The initial Helmholtz equation for a streamfunction can be represented in the plane case as [9]

$$\frac{\partial \Delta\psi}{\partial t} + J(\psi, \Delta\psi) = 0, \tag{1}$$

where the streamfunction is defined according to

$$v_x = -\frac{\partial\psi}{\partial y}, \qquad v_y = \frac{\partial\psi}{\partial x}\ ,$$

its Laplacian $\Delta\psi = \mathrm{curl}_z\,\mathbf{v}$ describes vorticity, and $J(\alpha,\beta)$ is Jacobian

$$J(\alpha,\beta) \equiv (\partial\alpha/\partial x)(\partial\beta/\partial y) - (\partial\beta/\partial x)(\partial\alpha/\partial y).$$

Eq. (1) represents the projection on the z axis of equation of vorticity conservation. In the plane two-dimensional case it looks as

$$\frac{d}{dt}\mathrm{curl}_z\,\mathbf{v} = 0, \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla. \tag{2}$$

Let us consider a radial flow and system of N point vortices in it. Then, following [5,10,11], the streamfunction can be represented as the sum of its regular ψ_{reg} and singular ψ_{sing} parts:

$$\psi = \psi_{\mathrm{reg}} + \psi_{\mathrm{sing}}, \quad \text{where } \psi_{\mathrm{sing}} = \frac{1}{2\pi}\sum_m A_m \ln|\mathbf{r} - \mathbf{r}_m|, \tag{3}$$

A_m is the mth vortex intensity (circulation), and r_m is its radius vector. The singular component satisfies the Poisson equation with the point sources in the right part

$$\Delta\psi_{\mathrm{sing}} = \sum_m A_m \delta(x - x_m)\delta(y - y_m). \tag{4}$$

Substituting (3) into (1) we obtain the equation for the regular component of the streamfunction

$$\frac{\partial \Delta\psi_{\mathrm{reg}}}{\partial t} + J(\psi_{\mathrm{sing}} + \psi_{\mathrm{reg}}, \Delta\psi_{\mathrm{reg}}) = 0 \tag{5}$$

and equations for the velocity components of the mth vortex [5]:

$$\dot{x}_m = -\frac{\partial(\psi_{\mathrm{reg}} + \psi^m_{\mathrm{sing}})}{\partial y}\bigg|_{\mathbf{r}=\mathbf{r}_m},$$
$$\dot{y}_m = \frac{\partial(\psi_{\mathrm{reg}} + \psi^m_{\mathrm{sing}})}{\partial x}\bigg|_{\mathbf{r}=\mathbf{r}_m}, \tag{6}$$

where ψ^m_{sing} is the streamfunction ψ_{sing} without the contribution of the mth vortex. We take the regular component ψ_{reg} in the form

$$\psi_{\mathrm{reg}} = -Q\varphi, \qquad Q = \mathrm{const} > 0. \tag{7}$$

It describes the radial divergent flow with source at the origin [5] with the velocity $V_r = Q/r$, $V_\varphi = 0$. Vorticity of the radial flow vanishes,

$$\Delta\psi_{\mathrm{reg}} = \frac{\partial^2\psi_{\mathrm{reg}}}{\partial r^2} + \frac{1}{r}\frac{\partial\psi_{\mathrm{reg}}}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi_{\mathrm{reg}}}{\partial\varphi^2} = 0$$

and the equation (5) becomes an identity.

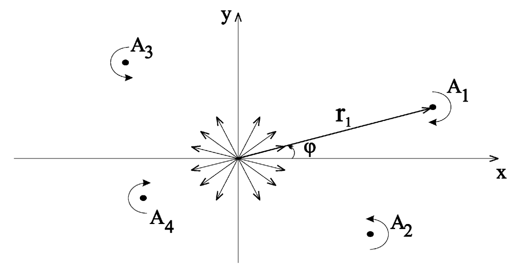

Fig. 1. Example of location of four vortices in a radial flow with a source at the origin.

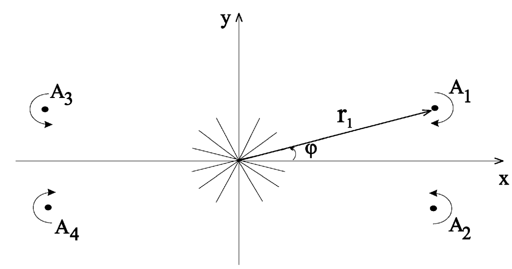

Fig. 2. Location of two vortex pairs in a radial flow with a source or a sink at the origin — symmetric case.

3. Symmetric system of four vortices

First, we consider the system of four vortices located arbitrarily relative to the center of radial flow (Fig. 1). It follows from the formula (3) that singular component of the streamfunction of one (1st) vortex moving under the influence of the flow and other vortices is

$$\psi^1_{\mathrm{sing}} = \frac{1}{2\pi}\big[A_2\ln|\mathbf{r}-\mathbf{r}_2| + A_3\ln|\mathbf{r}-\mathbf{r}_3| + A_4\ln|\mathbf{r}-\mathbf{r}_4|\big], \tag{8}$$

which, taking into account (6), results in the following motion equations for the 1st vortex

$$\dot{x}_1 = -\frac{1}{2\pi}\left[A_2\frac{y_{12}}{r_{12}^2} + A_3\frac{y_{13}}{r_{13}^2} + A_4\frac{y_{14}}{r_{14}^2}\right] + Q\frac{x_1}{r_1^2},$$
$$\dot{y}_1 = \frac{1}{2\pi}\left[A_2\frac{x_{12}}{r_{12}^2} + A_3\frac{x_{13}}{r_{13}^2} + A_4\frac{x_{14}}{r_{14}^2}\right] + Q\frac{y_1}{r_1^2}, \tag{9}$$

where $x_{12} = x_1 - x_2$, etc. Equations for other vortices are similar.

In the considered case of four vortices, forming two symmetric vortex pairs (Fig. 2), which imitate the motion of the dipole toroidal vortex, symmetry conditions are

$$A = A_3 = A_2 = -A_1 = -A_4,$$
$$|\mathbf{r}_1| = |\mathbf{r}_2| = |\mathbf{r}_3| = |\mathbf{r}_4| = |\mathbf{r}|. \tag{10}$$

This problem can be considered also as the problem of the head-on collision of equivalent vortex pairs in the presence of the flow. In the absence of the flow, the solution of this problem is well known: vortices remain all the time at vertexes of rectangle, whose sides change with time, scattering after all at right angle to the direction of initial motion. As a result of collision, the pairs "exchange" partners. Their exit velocity in absence of the flow is equal to the entrance velocity, as well as distances between the pair components [3,6].

Let us see how the flow influences on their motion. It is sufficient to consider equations of motion for only one vortex moving under the influence of three other vortices and the flow which, after taking into account the condition (10), are reduced to ($x \equiv x_1$, $y \equiv y_1$)