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Scattering of massless Dirac particles by oscillating barriers in one dimension

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ABSTRACT

We study the scattering of massless Dirac particles by oscillating barriers in one dimension. Using the Floquet theory, we find the exact scattering amplitudes for time-harmonic barriers of arbitrary shape. In all cases the scattering amplitudes are found to be independent of the energy of the incoming particle and the transmission coefficient is unity. This is a manifestation of the Klein tunneling in time-harmonic potentials. Remarkably, the transmission amplitudes for arbitrary sharply-peaked potentials also become independent of the driving frequency. Conditions for which barriers of finite width can be replaced by sharply-peaked potentials are discussed.

1. Introduction

Shortly after Dirac formulated his celebrated equation for relativistic electrons [1,2], Klein discovered that Dirac particles undergo anomalous tunneling at high potential barriers [3] (see Ref. [4] for a review). The classical example used to discuss the Klein tunneling is the potential step. When the potential exceeds the rest mass energy, low-energy electrons falling onto the potential step are always transmitted. As a consequence, a strong electrostatic barrier is not able to confine electrons to one side of it [5]. As pointed out by Sauter, this effect is quite independent of the potential profile and eventually depends only on its strength [6]. However, due to the unrealizable high potentials required, Klein tunneling was regarded as a curiosity in the context of relativistic quantum mechanics.

With the development of experimental methods to isolate graphene [7] came a renewed interest in Klein tunneling [8–10]. The reason for this interest is that electrons close to the Fermi energy can be described by the Dirac Hamiltonian for massless particles [11]. In this material Klein tunneling manifests itself as the occurrence of perfect transparency of barriers at normal incidence, as predicted by Katsnelson et al. [12,13] and later observed in experiments [14,15].

In this work we consider a massless Dirac particle moving in $1+1$ dimensions scattered by a time-dependent potential barrier. Using the Floquet theory we find the exact transmission amplitudes when the barrier is an arbitrary sharply-peaked function at the origin $x=0$, approaching the δ-function limit. We also study the transmission properties of massless Dirac electrons impinging on time-harmonic barriers of finite width and arbitrary shape, and the exact scattering amplitudes will be compared to those corresponding to a sharply-peaked function. The comparison will allow us to establish the conditions under which a finite-width barrier can be replaced by its δ-function limit. We conclude that the δ-function potential is a good approximation to more complex time-dependent barriers whose width is smaller than any other relevant length scale of the problem.

2. Time-harmonic sharply peaked barrier

In this section we study the scattering of massless Dirac particles by a time-dependent potential barrier of the form $V(x,t)=g(t)F(x)$. The spatial part of the potential $F(x)$ is assumed to be sharply peaked at the origin $x=0$, approaching the δ-function limit. Importantly, care must be taken when dealing with δ-function potentials in the Dirac equation [5,16,17]. The resulting equation is ambiguous if one takes the limit $F(x)\to\delta(x)$ from the outset. The origin of the ambiguity is the following. Since the Dirac equation is linear in momentum, the wave function itself must be discontinuous at $x=0$ to account for the singular δ-function potential. However, the product of a discontinuous function and the δ-function is mathematically ill defined. This ambiguity can be overcome by solving the corresponding Dirac equation for any arbitrary sharply peaked function and then taking the δ-function limit with the constraint $\int_{0^-}^{0^+} F(x)\,dx=1$ [5].

To obtain the proper boundary condition at the origin, we start with the $1+1$ massless Dirac equation

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$$i\dot{\boldsymbol{\psi}}(x,t)=\left[-i\sigma_x\frac{\partial}{\partial x}+V(x,t)\right]\boldsymbol{\psi}(x,t), \tag{1}$$

with $V(x,t)=g(t)F(x)$. The dot indicates the derivative with respect to time and the Pauli matrix σ_x acts on the two-component wave function $\boldsymbol{\psi}(x,t)$. We take units where the speed of light and $\hbar$ are equal to unity. The appropriate boundary condition for a time-independent sharply-peaked potential was obtained by McKellar and Stephenson in Ref. [5] We now generalize this approach for the time-dependent potential at hand. To this end we cast (1) in the form

$$\frac{\partial}{\partial x}\boldsymbol{\psi}(x,t)=\widehat{G}(x,t)\boldsymbol{\psi}(x,t), \tag{2a}$$

where

$$\widehat{G}(x,t)=-\sigma_x\left[\frac{\partial}{\partial t}+ig(t)F(x)\right]. \tag{2b}$$

Eq. (2a) is solved by a Neumann solution

$$\boldsymbol{\psi}(x,t)=\widehat{P}\exp\left[\int_{x_0}^{x}\mathrm{d}x'\,\widehat{G}\big(x',t\big)\right]\boldsymbol{\psi}(x_0,t), \tag{3}$$

where $\widehat{P}$ is the spatial ordering operator. Taking the limits $x\to 0^+$ and $x_0\to 0^-$, only the second term of the operator $\widehat{G}$, namely $-i\sigma_x g(t)F(x)$, contributes to the integral. This dominant term in the exponential commutes at spatially separated points, so we may set $\widehat{P}=1$. Recalling the constraint $\int_{0^-}^{0^+}F(x)\,\mathrm{d}x=1$ we finally arrive at the following boundary condition

$$\boldsymbol{\psi}\big(0^-,t\big)=\exp\big[ig(t)\sigma_x\big]\,\boldsymbol{\psi}\big(0^+,t\big). \tag{4}$$

In the absence of the potential term ($V(x,t)=0$), solutions of the massless Dirac equation (1) can be written as

$$\boldsymbol{\psi}_\pm(x,t)=\boldsymbol{\phi}_\pm e^{iE(\pm x-t)},\qquad \boldsymbol{\phi}_\pm=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}. \tag{5}$$

Notice that $\boldsymbol{\phi}_\pm$ are eigenvectors of σ_x since $\sigma_x\boldsymbol{\phi}_\pm=\pm\boldsymbol{\phi}_\pm$. Free solutions are then found to be plane waves traveling to the left or to the right, as deduced from the corresponding current density $j_\pm=\boldsymbol{\psi}_\pm^\dagger\sigma_x\boldsymbol{\psi}_\pm=\pm 1$.

We now turn to our main goal, the study of the effects of the oscillating barrier on the particle tunneling. The time dependence of the potential will be taken as $g(t)=g_0+g_1\cos\omega t$ hereafter. As in the case of the Schrödinger equation for an oscillating δ-function potential [18], the Floquet theory allows us to write the wave function in terms of the free solutions (5) as follows

$$\boldsymbol{\psi}(x,t)=\sum_{n=-\infty}^{\infty}\boldsymbol{A}_n(x)e^{-iE_nt},\quad x\neq 0, \tag{6a}$$

where $E_n=E+n\omega$, E is a quasi-energy and n is the sideband channel index. Since we are interested in electron transmission across the barrier, we take the following ansatz for the spinors $\boldsymbol{A}_n(x)$ in the expansion (6a)

$$\boldsymbol{A}_n(x)=i^n\times\begin{cases}\delta_{n0}e^{iE_nx}\boldsymbol{\phi}_+ + r_ne^{-iE_nx}\boldsymbol{\phi}_-, & x<0,\\ t_ne^{iE_nx-ig_0}\boldsymbol{\phi}_+, & x>0.\end{cases} \tag{6b}$$

The first phase factor i^n is introduced for later convenience. Furthermore, the second phase factor $\exp(-ig_0)$ will cancel the term $\exp(i\sigma_xg_0)$ after applying the boundary condition (4) since $\exp(i\sigma_xg_0)\boldsymbol{\phi}_+=\exp(ig_0)\boldsymbol{\phi}_+$. These two phase factor have no effect on the transmission probabilities $|t_n|^2$ but will simplify the final expression of the transmission amplitudes.

It is straightforward to calculate the time-averaged current density of the wave function (6)

$$\langle j\rangle=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\boldsymbol{\psi}(x,t)^\dagger\sigma_x\boldsymbol{\psi}(x,t)\,\mathrm{d}t=\begin{cases}1-R, & x<0,\\ T, & x>0,\end{cases} \tag{7a}$$

where

$$R=\sum_{n=-\infty}^{\infty}|r_n|^2,\qquad T=\sum_{n=-\infty}^{\infty}|t_n|^2, \tag{7b}$$

are the reflection and transmission probabilities, respectively.

Inserting the ansatz (6) in Eq. (4), multiplying by $\exp(iE_mt)$, m being an arbitrary integer, and time-averaging over one period we get

$$\delta_{m0}\boldsymbol{\phi}_+ + r_m\boldsymbol{\phi}_-=\sum_{n=-\infty}^{\infty}J_{m-n}(g_1)t_n\boldsymbol{\phi}_+,\quad m=0,\pm 1,\ldots, \tag{8}$$

where $J_\ell(z)$ denotes the Bessel function of the first kind. After multiplying from the left by $\boldsymbol{\phi}_-^{\mathsf{T}}$ we conclude that $r_m=0$. Consequently, the reflection probability R vanishes. From (8) with $r_m=0$, and recalling the orthonormality condition of Bessel functions, one finally gets

$$t_n=J_{-n}(g_1),\quad n=0,\pm 1,\ldots, \tag{9}$$

which satisfies $T=\sum_{n=-\infty}^{\infty}|t_n|^2=1$, as expected from the previous result $R=0$.

Several important conclusions can be drawn from the above results. First, the transmission probability is always unity, indicating that Klein tunneling persists even if the barrier is harmonically modulated in time ($g_1\neq 0$). Nevertheless, the transmission probability through the elastic channel $T_0=|t_0|^2=J_0^2(g_1)<1$ is reduced as compared to the static barrier, for which $T_0=1$. Second, the transmission amplitudes given by (9) are independent of the quasi-energy E and the driving frequency ω. We will see in the next section that the latter is a consequence of the peculiarities of having an infinitely narrow barrier.

3. Time-harmonic square barrier

We now consider the scattering from a square barrier of finite width a. In this situation, the potential appearing in the massless Dirac equation (1) is

$$V(x,t)=\begin{cases}V_0+V_1\cos\omega t, & -a/2<x<a/2,\\ 0, & \text{otherwise}.\end{cases} \tag{10}$$

Solutions of the Dirac equation (1) with the time-harmonic potential (10) can be again written down as in Eq. (6a), now including the origin of coordinates $x=0$ since the potential is nonsingular everywhere. Outside the barrier region ($|x|>a/2$) the spinors $\boldsymbol{A}_n(x)$ are expressed as combination of traveling waves

$$\boldsymbol{A}_n(x)=i^n\times\begin{cases}\delta_{n0}e^{i(E_nx+V_0a/2)}\boldsymbol{\phi}_+ + r_ne^{-i(E_nx+V_0a/2)}\boldsymbol{\phi}_-,\\ \quad x<-a/2,\\ t_ne^{i(E_nx-V_0a/2)}\boldsymbol{\phi}_+,\\ \quad x>a/2,\end{cases} \tag{11a}$$

where the phase factors i^n and $\exp(\pm iV_0a/2)$ are introduced for later convenience. Inside the barrier region, solutions are of the form (see, e.g., Refs. [19–21])

$$\boldsymbol{A}_n(x)=\sum_{p=-\infty}^{\infty}\big[A_pe^{i(E_p-V_0)x}\boldsymbol{\phi}_+ + B_pe^{-i(E_p-V_0)x}\boldsymbol{\phi}_-\big]\times J_{n-p}(V_1/\omega), \tag{11b}$$

where $x<|a/2|$.