Physics Letters A 378 (2014) 937–940

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Demonstrating nonlocality-induced teleportation through Majorana bound states in a semiconductor nanowire

Peiyue Wang [a], Yunshan Cao [b,c], Ming Gong [d], Shu-Shen Li [e], Xin-Qi Li [a,*]

[a] Department of Physics, Beijing Normal University, Beijing 100875, China
[b] School of Physics, Peking University, Beijing 100871, China
[c] Kavli Institute of NanoScience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
[d] Department of Physics and Centre for Quantum Coherence, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, China
[e] State Key Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, P.O. Box 912, Beijing 100083, China

ARTICLE INFO

Article history:
Received 20 July 2013
Received in revised form 14 January 2014
Accepted 20 January 2014
Available online 4 February 2014
Communicated by P.R. Holland

Keywords:
Majorana fermion
Nonlocality
Teleportation

ABSTRACT

It was predicted by Tewari et al. (2008) [15] that a *teleportation-like* electron transfer phenomenon is one of the novel consequences of the existence of Majorana fermion, because of the inherently nonlocal nature. In this work we consider a concrete realization and measurement scheme for this interesting behavior, based on a setup consisting of a pair of quantum dots which are tunnel-coupled to a semiconductor nanowire and are jointly measured by two point-contact detectors. We analyze the teleportation dynamics in the presence of measurement back-action and discuss how the teleportation events can be identified from the current trajectories of strong response detectors.

The search for Majorana fermions in solid states has been attracting a great deal of attention in the past years [1–8]. In solid states, it has been predicted that the Majorana bound states (MBSs) can appear for instance in the 5/2 fractional quantum Hall system [9] and the p-wave superconductor and superfluid [10]. In particular, an effective p-wave superconductor can be realized by a semiconductor nanowire with Rashba spin–orbit interaction and Zeeman splitting and in proximity to an s-wave superconductor [3–6]. This opens a new avenue of searching for Majorana fermions using the most conventional materials. Also, some demonstrating schemes were proposed based on various transport signatures, including the tunneling spectroscopy which may reveal characteristic zero-bias conductance peak [11,12] and peculiar noise behaviors [13,14], the nonlocality nature of the MBSs [15,16], and the 4π periodic Majorana–Josephson currents [1–3,17]. In the aspect of experiment, exotic signatures that may reveal the existence of MBSs have been observed in the system of semiconductor nanowire in proximity to an s-wave superconductor [18–21].

An inevitable consequence of the existence of Majorana zero modes is that the fermion quasiparticle excitations are inherently nonlocal. To be specific, let us consider a semiconductor nanowire in the topological regime which thus supports the MBSs at the two ends [3,5,6,18], and denote the MBSs by Majorana operators γ_1 and γ_2. They are related to the regular fermion operator in terms of $f^\dagger = (\gamma_1 + i\gamma_2)/\sqrt{2}$ and its Hermitian conjugate f. This connection implies some remarkable consequences. For instance, if an electron with energy smaller than the energy gap between the Majorana zero mode and other exited states is injected into the system, we can only have the excitation described by f and $f^\dagger$. This means that a single electron is "split" into two Majorana bound states which are, however, spatially separated. In this work, instead of exploiting certain *indirect* transport signatures, we discuss a possible and very *direct* way to demonstrate this intrinsic *nonlocality* of the paired Majorana modes.

The proposed scheme is schematically displayed in Fig. 1, where the two MBSs, generated at the ends of the nanowire, are tunnel-coupled to two quantum dots (QDs), respectively. Moreover, the QDs are jointly probed by the nearby quantum-point-contact (QPC) detectors. This proposal is motivated by the nowadays state-of-the-art technique, which enables the QPC current to sensitively probe an extra single electron in the nearby quantum dot [22]. In Ref. [15], an equivalent "dot-MBSs-dot" system is analyzed by assuming an extra electron initially in one of the QDS and considering its transmission through the MBSs in a vanished hybridization limit. Corresponding to the nanowire realization in Fig. 1, their prediction indicates that, in a "long-wire" limit, the electron can transmit through the nanowire on a finite (short) timescale, revealing thus a "teleportation" phenomenon. In our present work, following Ref. [15], we call this *ultrafast* transfer behavior *teleportation*, which is actually a remarkable consequence of Majorana's

* Corresponding author. Tel.: +86 10 58801874.
E-mail address: lixinqi@bnu.edu.cn (X.-Q. Li).

http://dx.doi.org/10.1016/j.physleta.2014.01.039
0375-9601/© 2014 Published by Elsevier B.V.

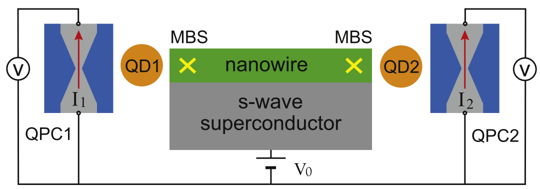

Fig. 1. Schematic setup of using two point-contact detectors to demonstrate the Majorana-nonlocality-induced *teleportation-like* electron transfer between two remote quantum dots. The semiconductor nanowire is in contact with an *s*-wave superconductor, so that under appropriate conditions a pair of Majorana bound states (MBS) are anticipated to appear at the ends of the nanowire. Here we show the schematic closed circuit, in which the chemical potential of the superconductor and the bias voltages across the detectors are explicitly defined.

nonlocality. Related to the scheme of joint-measurements shown in Fig. 1, we will carry out the teleportation dynamics under the influence of measurement back-action, and discuss how the teleportation events can be identified from the current trajectories of strong response detectors.

1. Model

The setup of Fig. 1 can be described by the following Hamiltonian

$$H = H_{sys} + H_{pc}. \tag{1}$$

The *system* Hamiltonian, H_{sys}, describes the MBSs plus the single-level QDs and their tunnel coupling as follows [2,12–15]

$$H_{sys} = i\frac{\epsilon_M}{2}\gamma_1\gamma_2 + \sum_{j=1,2}\left[\epsilon_j d_j^\dagger d_j + \lambda_j\left(d_j^\dagger - d_j\right)\gamma_j\right]. \tag{2}$$

Here γ_1 and γ_2 are the Majorana operators associated with the two MBSs at the ends of the nanowire. The two MBSs interact with each other by a strength $\epsilon_M \sim e^{-L/\xi}$, which damps exponentially with the length (L) of the nanowire, with a characteristic length of the superconducting coherent length (ξ). $d_1(d_1^\dagger)$ and $d_2(d_2^\dagger)$ are the annihilation (creation) operators of the two single-level quantum dots, while λ_1 and λ_2 are their coupling amplitudes to the MBSs. In practice, it will be convenient to switch from the Majorana representation to the regular fermion one, through the transformation of $\gamma_1 = i(f - f^\dagger)$ and $\gamma_2 = f + f^\dagger$. We can easily check that f and $f^\dagger$ satisfy the anti-commutative relation, $\{f, f^\dagger\} = 1$. After an additional local gauge transformation, $d_1 \to id_1$, we reexpress Eq. (2) as

$$H_{sys} = \epsilon_M\left(f^\dagger f - \frac{1}{2}\right) + \sum_{j=1,2}\left[\epsilon_j d_j^\dagger d_j + \lambda_j\left(d_j^\dagger f + f^\dagger d_j\right)\right]$$
$$- \lambda_1\left(d_1^\dagger f^\dagger + f d_1\right) + \lambda_2\left(d_2^\dagger f^\dagger + f d_2\right). \tag{3}$$

It should be noticed that the tunneling terms in this Hamiltonian only conserve charge modulo $2e$. This reflects the fact that a pair of electrons can be extracted out from the superconductor condensate and can be absorbed by the condensate.

The other Hamiltonian in Eq. (1), H_{pc}, is for the two point-contacts which reads

$$H_{pc} = \sum_{j=1,2}\sum_{l_j,r_j}\left[\left(\varepsilon_{l_j} c_{l_j}^\dagger c_{l_j} + \varepsilon_{r_j} c_{r_j}^\dagger c_{r_j}\right) + \left(w_j c_{l_j}^\dagger c_{r_j} + \text{H.c.}\right)\right]. \tag{4}$$

This Hamiltonian simply describes electron tunneling through a potential barrier between two electronic reservoirs (with electron creation and annihilation operators, $c_{l_j(r_j)}^\dagger$ and $c_{l_j(r_j)}$). We assume that the tunneling amplitudes (w_j) are approximately of energy independence. Thus w_j does not depend on the associated states "l_j" and "r_j". However, in w_j we should include the effect of the nearby quantum dot, since its occupation would change the tunneling amplitudes. We account for this effect in terms of $w_j = \Omega_j + \Delta\Omega_j d_j^\dagger d_j$.

2. Teleportation

Let us consider the transfer problem of an *extra* electron between the two quantum dots, which is assumed initially in the left quantum dot.

In this part we assume a simpler setup in the absence of the point-contact detectors [15].

In particular, we consider the weak interaction limit $\epsilon_M \to 0$, in order to reveal the remarkable *teleportation* behavior. Using the transformed representation, $|n_1, n_M, n_2\rangle$ describes the possible charge configuration of the dot-MBSs-dot system, where $n_{1(2)}$ and n_M denote, respectively, the electron number ("0" or "1") in the left (right) dot and the central MBSs. Totally, we have eight basis states, which can be divided into two subspaces: $|100\rangle, |010\rangle, |001\rangle, |111\rangle$ with odd parity (electron numbers); and $|110\rangle, |101\rangle, |011\rangle, |000\rangle$ with even parity. Associated with our specific initial condition, we will only have the odd-parity states involved in the state evolution. Moreover, for simplicity, we assume $\lambda_1 = \lambda_2 = \lambda$ and $\epsilon_1 = \epsilon_2 = 0$ throughout this work.

Simple calculation can give the occupation probabilities of the left and right dots, respectively, as $P_1(t) = \cos^2(\lambda t)$ and $P_2(t) = \sin^2(\lambda t)$. Here, for each of the probabilities, it contains two possible occupations: $|100\rangle$ and $|111\rangle$ for $P_1(t)$; $|001\rangle$ and $|111\rangle$ for $P_2(t)$. Now, we introduce (extract) the partial probability $P_2^{(1)}(t) = |\langle 001|e^{-iH_{sys}t}|100\rangle|^2$ from $P_2(t)$, which has also a simple form, $P_2^{(1)}(t) = \sin^4(\lambda t)$. Similarly, we may define $P_2^{(2)}(t) = |\langle 111|e^{-iH_{sys}t}|100\rangle|^2$, which can be obtained simply by $P_2^{(2)}(t) = P_2(t) - P_2^{(1)}(t) = \sin^2(\lambda t)\cos^2(\lambda t)$. Based on these simple manipulations, of great interest is the result of $P_2^{(1)}(t)$, since it implies that, even in the limit of $\epsilon_M \to 0$ (very "long" nanowire), the electron in the left dot can transmit through the MBSs and appear in the right dot on some finite (short) timescale. This is the remarkable "teleportation" phenomenon discussed in Ref. [15] which, surprisingly, holds a "superluminal" feature. In the following, to prove this teleportation behavior, we propose to use QPC detectors to perform a *coincident* measurement of both the occupation numbers of the left and the right dots. This type of measurement can distinguish the process responsible for $P_2^{(1)}(t)$ from that responsible for $P_2^{(2)}(t)$.

3. Demonstration

Now we turn to the measurement setup of Fig. 1. Physically, the measurements will cause back-action on the charge transfer dynamics in the central dot-MBSs-dot system. This effect can be described by a master equation, formally expressed as [23]

$$\dot{\rho} = -i\mathcal{L}\rho - \mathcal{R}\rho. \tag{5}$$

The first term denotes $\mathcal{L}\rho = [H_{sys}, \rho]$, and the second term describes the measurement back-action. More specifically, $\mathcal{R}\rho = \frac{1}{2}\sum_{j=1,2}\{[w_j^\dagger, \tilde{w}_j^{(-)}\rho - \rho\tilde{w}_j^{(+)}] + \text{H.c.}\}$, where $\tilde{w}_j^{(\pm)} = C_j^{(\pm)}(\pm\mathcal{L})w_j$. $C_j^{(\pm)}(\pm\mathcal{L})$ are the Liouvillian counterparts of the QPC spectral functions $C_j^{(\pm)}(\pm\omega)$, which were obtained explicitly in Ref. [23]. In this work, we restrict to a wideband limit and large bias condition for the point-contact detectors, which allow us to approximate $C_j^{(\pm)}(\pm\mathcal{L})$ by $C_j^{(\pm)}(0)$. More explicitly, we have [23]: $C_j^{(\pm)}(0) = \pm 2\pi g_L g_R eV_j/(1 - e^{\mp\beta eV_j})$, where $g_{L(R)}$ is the density-of-states