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Quantum correlations support probabilistic pure state cloning

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ABSTRACT

The probabilistic scheme for making two copies of two nonorthogonal pure states requires two auxiliary systems, one for copying and one for attempting to project onto the suitable subspace. The process is performed by means of a unitary-reduction scheme which allows having a success probability of cloning different from zero. The scheme becomes optimal when the probability of success is maximized. In this case, a bipartite state remains as a free degree which does not affect the probability. We find bipartite states for which the unitarity does not introduce entanglement, but does introduce quantum discord between some involved subsystems.

1. Introduction

Quantum correlations between two systems are well characterized by entanglement of formation (EOF) and quantum discord [1,2]. Both become indispensable ingredients in some quantum information protocols. For instance, the deterministic and the probabilistic teleportation schemes require a pure entangled state as channel [3,4], if one looks for a result with fidelity 1. Protocols such as quantum non-locality and quantum search are realizable even without entanglement but with nonclassical separable states [5]. In addition, the optimal process for unambiguous state discrimination can be performed without entanglement but with quantum discord only [6]. Now we can understand that the discord is a quantum correlation measurement that gives account for the quantumness not captured by entanglement [7]. In this context, an interesting and fundamental process to be researched is the probabilistic quantum cloning mechanism. The linearity of quantum theory forbids the perfect copying of an unknown quantum state [8,10,11]. However, a known set of orthogonal pure states can be cloned deterministically [8] while a linearly independent (LI) set of known nonorthogonal pure states can be cloned only probabilistically [12–14]. The probabilistic schemes clone the states with fidelity 1 when the process is successful, which becomes an advantage over the deterministic scheme. Each of these schemes must be assisted by auxiliary quantum systems and they must be coupled by means of a joint unitary operation. This kind of unitary demands a direct interaction between the systems and an effective control on the interaction Hamiltonian of them. For instance, the basic control not gate for two cold trap ions can be accomplished by increasing the Hilbert space dimension using vibrational freedom degrees [15]. Since entanglement cannot be generated with local operations and classical communication (LOCC) [16] it is considered as a resource. On the other hand, quantum discord can be generated by certain LOCC [17]; accordingly it is a resource when we are restricted only to local unitary operations. Additionally, we have shown [18] that the generation of a high quantum correlation degree requires nonconservative Hamiltonian interaction terms. Therefore, by removing the unnecessary correlations in a particular protocol, the unitary operations and therefore the quantum interactions for achieving the process are reduced or simplify as well. Thus, natural questions arising in this respect: what are the quantum correlations that underlie the probabilistic cloning scheme? and can the process be achieved without some of them? In this article we analyze entanglement [1] and quantum discord [2] underlying the scheme for probabilistic cloning of two known nonorthogonal pure states with maximum probability of success. In particular we study these correlations between two parties and between two subsystems. At this respect, an interesting analysis of the entanglement structure contained in the deterministic cloning machine was realized in Ref. [9].

This article is organized as follows: In Section 2 we present an overview of basic results concerning a probabilistic quantum cloning scheme. In Section 3 we find the conditions which fix the unitary transformation parameters for performing the quantum cloning with maximal probability of success and without EOF in some partitions. In the last section we summarize our results.

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2. The 1 → *m* probabilistic cloning

In this section we succinctly describe the probabilistic $1 \to m$ cloning scheme [12–14]. It is necessary to have complete control over $m+1$ qubits; the *original* one denoted by the subindex o, where one of two known states $\{|\alpha_1\rangle_o, |\alpha_2\rangle_o\}$ is randomly prepared, $(m-1)$ qubits labeled by the subindex c where $m-1$ copies, $\{|\alpha_k\rangle_c^{\otimes(m-1)}\}$ could be printed, and an ancillary qubit labeled by the subindex a is introduced in order to perform a von Neumann measurement process on it. We consider two LI states $\{|\alpha_1\rangle_o, |\alpha_2\rangle_o\}$ prepared with a priori probabilities $\{\eta_1, \eta_2\}$ respectively ($\eta_1+\eta_2=1$). The c and a systems are initially in the known pure state $|\Omega\rangle_{c,a}$. Besides, we assume that there exists a unitary operation U, which can perform the two following transformations:

$$\begin{aligned}|\Lambda_1\rangle &= U|\alpha_1\rangle_o|\Omega\rangle_{c,a} \\ &= \sqrt{p_1}|\alpha_1\rangle_{o,c}^{\otimes m}|\tau_s\rangle_a + \sqrt{1-p_1}|\chi\rangle_{o,c}|\tau_f\rangle_a, \qquad (1)\\ |\Lambda_2\rangle &= U|\alpha_2\rangle_o|\Omega\rangle_{c,a} \\ &= \sqrt{p_2}|\alpha_2\rangle_{o,c}^{\otimes m}|\tau_s\rangle_a + \sqrt{1-p_2}|\chi\rangle_{o,c}|\tau_f\rangle_a, \qquad (2)\end{aligned}$$

where $\{|\tau_s\rangle_a, |\tau_f\rangle_a\}$ is an orthonormal basis and the integer $m \geqslant 2$ is the number of copies. Here we have considered the tensorial product notation $|\alpha_k\rangle_{o,c}^{\otimes m} = |\alpha_k\rangle_o \otimes |\alpha_k\rangle_{c_1} \otimes |\alpha_k\rangle_{c_2} \otimes \cdots \otimes |\alpha_k\rangle_{c_{m-1}}$. The $1 \to m$ cloning attempt is achieved when a von Neumann measurement projects the a system onto the $|\tau_s\rangle_a$ state. This holds with probability

$$P_s = \eta_1 p_1 + \eta_2 p_2, \qquad (3)$$

otherwise the process fails, i.e., when the a system is projected onto the $|\tau_f\rangle_a$, which has a probability $P_f = 1 - P_s$. In that case the o and the c systems collapse into the normalized state $|\chi\rangle_{o,c}$ whatever be the initial state $|\alpha_k\rangle$.

Since a unitary operator preserves the overlap between states, from Eqs. (1) and (2) we obtain the constraint

$$\alpha = \alpha^m \sqrt{p_1 p_2} + \sqrt{(1-p_1)(1-p_2)}, \qquad (4)$$

which has to be satisfied by the parameters p_1, p_2, m and $\alpha = \langle\alpha_1|\alpha_2\rangle$. Without loss of generality, we always can consider the overlap α to be real by managing the axes on the Bloch sphere properly. From Eq. (4) we can express p_2 as a function of p_1, finding the two following solutions:

$$p_{2,\pm} = \frac{(\alpha^{m+1}\sqrt{p_1} \pm \sqrt{[1-\alpha^2-(1-\alpha^{2m})p_1](1-p_1)})^2}{[1-(1-\alpha^{2m})p_1]^2}, \qquad (5)$$

where $0 \leqslant p_1 \leqslant 1$ ensures that p_2 is also a real number between 0 and 1. Inserting $p_{2,\pm}$ from Eq. (5) into Eq. (3) we can find the maximum value of the success probability P_s with respect to p_1. The value of p_1 for which P_s reaches its highest value, in general, can be found numerically. In the special cases of equal a priori probabilities $\eta_1 = \eta_2$, the maximum success probability of cloning is reached at $p_1 = p_2 = (1-\alpha)/(1-\alpha^m)$ and it becomes

$$P_{s,\max} = \frac{1-\alpha}{1-\alpha^m}. \qquad (6)$$

It follows from the above expression that the maximum probability becomes the Duan and Guo result for $m = 2$ [12]. Note that as m rises the $P_{s,\max}$ decreases and asymptotically goes to the probability of unambiguous state discrimination in the limit $m \to \infty$ [19]. Therefore, with respect to the number of copies m, the maximal probability of success holds for the attempt to obtain two copies.

In what follows we study the EOF and the quantum discord between the involved systems in the process with maximum probability for attaining two copies of two nonorthogonal states prepared with equal a priori probabilities.

3. Quantum correlations

Quantum correlations between two systems are well characterized by the EOF [1] and the quantum discord [2]. The amount of EOF coincides with the value of the quantum discord for bipartite pure states [7] while they, in general, are different for mixed states. For instance, nonclassical separable states lack entanglement and have discord different from zero. However, from the geometrical point of view [7] one easily deduces that in an entangled mixed state the quantum discord cannot be absent. The entanglement of a mixed state of two qubits can be evaluated by means of the concurrence which is a monotone function with the EOF [1]. The concurrence allows obtaining the EOF which is an entropy based measurement and therefore the amount of EOF can be compared to the quantum discord values. For evaluating EOF and quantum discord we used the Koashi–Winter identity [20], and we follow the recipe given in Ref. [21]. That recipe allows obtaining numerically or analytically those correlations for a bipartite system of $2 \otimes d$ dimension, with $d \leqslant 2$ when the mixed state is of rank 2. Additionally, in some cases we make use of the partially transposed criterion [22] in order to discard the evaluation of the EOF when appropriate. We evaluate the quantum discord only in the partition where EOF is absent.

As we saw above, the probabilistic cloning scheme requires the unitary transformation (2), which must have the ability of mapping the two different factorized states $|\alpha_k\rangle|\Omega\rangle$ onto the two nonfactorized ones $|\Lambda_k\rangle$. Each $|\Lambda_k\rangle$ reveals, in principle, the presence of EOF and discord between the involved subsystems. However the quantum correlations involved in this process are those of the density operator

$$\rho_U = \eta_1|\Lambda_1\rangle\langle\Lambda_1| + \eta_2|\Lambda_2\rangle\langle\Lambda_2|. \qquad (7)$$

In what follows we focus on the scheme with maximum probability of obtaining two copies ($m=2$), where the two possible states $\{|\alpha_k\rangle\}$ are prepared with equal a priori probability ($\eta_1 = \eta_2 = 1/2$). In this case $P_{s,\max} = p_1 = p_2 = 1/(1+\alpha)$. Note that the transformation U in Eqs. (1) and (2) has p_1 and $|\chi\rangle$ as two adjustable quantities. We have found the maximum probability for achieving the goal of cloning by fixing the p_1 parameter, however, neither of the probabilities of success P_s nor $P_{s,\max}$ depends on the $|\chi\rangle$ state. In other words, the demand for a maximum success probability fixes the parameter p_1 and does not impose conditions on the state $|\chi\rangle$. The $|\chi\rangle$ state inevitably appears with probability $1 - P_{s,\max} = \alpha/(1+\alpha)$ different from zero. Here we are looking for particular $|\chi\rangle$ with the aim of removing the EOF between some partitions.

After numerous numerical calculations we realized that there are three states $|\chi\rangle$ for which EOF becomes zero between for some possible partitions. Those special $|\chi\rangle$ are associated with three symmetries defined with respect to the two states $|\alpha_1\rangle|\alpha_1\rangle$ and $|\alpha_2\rangle|\alpha_2\rangle$. The first considered $|\chi_{s-E}\rangle$ is symmetrically located between the two states $|\alpha_1\rangle|\alpha_1\rangle$ and $|\alpha_2\rangle|\alpha_2\rangle$. It is the symmetric entangled state

$$|\chi_{s-E}\rangle = \frac{|\alpha_1\rangle|\alpha_1\rangle + |\alpha_2\rangle|\alpha_2\rangle}{\sqrt{2(1+\alpha^2)}}. \qquad (8)$$

The second one is orthogonal to the subspace defined by $|\alpha_1\rangle|\alpha_1\rangle$ and $|\alpha_2\rangle|\alpha_2\rangle$, say one of the factorized states

$$|\chi_\perp\rangle = |\alpha_{1\perp}\rangle|\alpha_{2\perp}\rangle \quad \text{or} \quad |\chi_\perp\rangle = |\alpha_{2\perp}\rangle|\alpha_{1\perp}\rangle \qquad (9)$$

where $|\alpha_{1\perp}\rangle$ and $|\alpha_{2\perp}\rangle$ are the biorthogonal states of $|\alpha_1\rangle$ and $|\alpha_2\rangle$ respectively, i.e., $\langle\alpha_1|\alpha_{1\perp}\rangle = 0$ and $\langle\alpha_2|\alpha_{2\perp}\rangle = 0$. The third considered $|\chi\rangle$ is symmetrically put between two states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ of each qubit o and c, i.e., the factorized state