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Time scales in evolutionary game on adaptive networks

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ABSTRACT

Most previous studies concerning spatial games have assumed strategy updating occurs with a fixed ratio relative to interactions. We here set up a coevolutionary model to investigate how different ratio affects the evolution of cooperation on adaptive networks. Simulation results demonstrate that cooperation can be significantly enhanced under our rewiring mechanism, especially with slower natural selection. Meanwhile, slower selection induces larger network heterogeneity. Strong selection contracts the parameter area where cooperation thrives. Therefore, cooperation prevails whenever individuals are offered enough chances to adapt to the environment. Robustness of the results has been checked under rewiring cost or varied networks.

1. Introduction

Cooperation is a central feature of biological and social systems [1,2]. However, the emergence and persistence of cooperation remains an evolutionary puzzle, because cooperator benefits others at a personal cost, which is apparently in contradiction to the basic premise of natural selection, and thus would be weeded out in the evolution. Evolutionary game theory [3–5] has provided a powerful framework to model and address problems surrounding this issue. In the setup of evolutionary game, subpopulations differing in behaviors or phenotypes are discriminated in game strategies. Individuals interact according to game rules, collecting payoffs that reveal their personal fitness in evolution. And the results reflect the evolution outcomes. Up to now, a number of mechanisms supporting cooperation have been proposed, including kin selection [6], group selection [7], direct reciprocity [8,9], indirect reciprocity [10–12] and network reciprocity [13]. In particular, network reciprocity has received intensive studies [14–27]. Different from well mixed scenarios, where for each individual it is equally possible to interact with anyone else, in spatial games, where network connections denote links between individuals, interactions are restricted to be only with direct neighbors. In this case, cooperation can be maintained owing to spatial clusters. Apart from regular lattices [13,14], games on other networks are also widely investigated, such as random [16], small-world [19], and scale-free networks [15,17,22]. The effect of varying network characteristics on cooperation are also explored, including clustering [20,26], community structure [21], or other realistic properties [18,23,24].

Recently, the coevolution of strategy and network structure receives increasing attention [28–41]. In these settings, not only the strategies of individuals are evolving, but also the underlying network of social interactions. The changes in environment feed back to the strategy evolution. This constitutes a natural upgrade of the evolutionary spatial games, because it better captures the ever-changing features of the underlying social networks. Moreover, individuals seek to adjust their social ties in pursuit of better well-being. The changes in networks of interactions usually take form of the population growth [28–30], the rewiring of links [32–36], or mobility [37–41]. These coevolution rules have been shown to greatly promote or impact the evolution of cooperation.

In games played on networks, it is traditionally assumed that, before the strategy updating event occurs, each individual interacts with all of its direct neighbors. This implies that selection happens at a much slower rate than that of interaction. However, this is not always the case in biological context [42,43]. Theoretical work in well mixed population has also shown that releasing the constraint of slow selection, i.e., varying time scales, brings dramatic changes to evolutionary outcomes [44]. Therefore, it is intriguing to investigate how time scale of selection to interaction could affect cooperation in spatial games, especially in coevolutionary games with adaptive networks. Motivated by this, we propose a simple coevolution model to explore the effect of time scales on cooperation in adaptive networks. In each generation, a tunable

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number of partner pairs are picked up to interact. Then those players, who are dissatisfied with their partners, can terminate the links and rewire to random ones [45]. After that, strategy updating occurs synchronously for all individuals. The number of chosen individuals for interaction covers the range of time scales from the fastest selection to the slowest one. By Monte Carlo simulations, we demonstrate that under the rewiring mechanism in pair-wise interactions, cooperation can be significantly enhanced, especially with slower natural selection. We have also investigated the influence of selection intensity, and find that strong selection restricts the temptation parameter area where cooperation is predominant for any time scale. Therefore, cooperation prevails when individuals are offered sufficient chances to adjust their social ties.

The rest of the paper is organized as follows: In Section 2, detailed description of our model is given. In Section 3, we present the results of simulations along with the discussions. Finally, we arrive at the conclusion in Section 4.

2. Model

In this work, the famous prisoner's dilemma game (PDG) is employed as the metaphor for the competing interests between individual and group. For a typical PDG, each of the two players either cooperates or defects. They both receive a reward R upon mutual cooperation and punishment P upon mutual defection. When confronted with a cooperator, the defector gains a temptation to defect T, while the exploited cooperator acquires a sucker's payoff S. The ranking of the four payoff values satisfies $T > R > P > S$. Following common practices [13,35], we rescale the payoff matrix as

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 0 \\ 1+u & u \end{pmatrix} \end{array}$$

such that the game is controlled by a single parameter $u \in (0, 1)$, which indicates the advantage of defection over cooperation when facing a cooperator, i.e., the temptation to defect.

Initially, the network of social contacts starts from an ER random network, where N individuals located on the sites of network are randomly paired up by M links. Each individual has the same probability (0.5) to be a cooperator (C) or a defector (D). Different from the node-based updating mechanism commonly adopted in previous studies [14–17,19], here we focus on the links between individuals. In each generation, a number of links are randomly picked up. These pairs of individuals at the end of the links play PDGs with their partners. Then one player in each pair has the chance to break the link and rewire to a new one randomly [35]. Each individual stores a cumulated payoff. After all these pairs have completed the game playing and rewiring, strategies are updated synchronously for the whole population. Each individual i imitates the strategy of a randomly chosen neighbor j with a probability given by Fermi function

$$\phi(s_i \leftarrow s_j) = \frac{1}{1 + \exp[\beta(P_i - P_j)]}$$

where $\beta \in [0, \infty)$ denotes the selection intensity. $\beta \to 0$ leads to random drift, while $\beta \to \infty$ leads to the deterministic imitation dynamics.

To specify the time scale of interaction to selection, we introduce a normalized parameter $s \in [0, 1]$ such that the real number of pairs chosen for interaction is given by $s \cdot k \cdot N/2$, where k is the average degree of the network. $s \to 0$ corresponds to very fast selection, while $s \to 1$ recovers the traditional mode that each individual averagely interacts with each of its neighbors for once before strategy updating, i.e., a slow natural selection.

For the rewiring of links, one node in the pair is randomly chosen to terminate the link to the other node with a probability P_c (P_d) if the other node is a C (D), and to pick a player from the remainder as its new partner. In this manner the total number of links, or equivalently, the average degree of the network remains unchanged during coevolution. Averagely, the CC, CD, and DD pairs are severed with probabilities P_c, $(P_c + P_d)/2$, and P_d, respectively. For the specific value of P_c and P_d, we will first adopt the empirical data in human experiment [45], and then extend to the full parameter space. Note that cooperators do not have the priority over defectors in link rewiring chances. Therefore the isolated defectors induced by this prescribed bias [46] do not appear in this model.

A key quantity that specifies the behavior of the system is the fraction of cooperators f_c after the equilibrium state has been reached. Central issues concerning our interests include effects of varying time scales and the selection intensity on the evolution of cooperation. In the latter part of this paper, we extend our model to more realistic cases where cost in link rewiring exists. Robustness of the results is also checked by adopting alternative initial networks.

3. Results

3.1. *Effect of time scales on cooperation*

In what follows, we present the results of simulations. As a reference, we will first consider the evolution of cooperation in the absence of link rewiring. The interplay of temptation to defect u and time scale s is illustrated in a contour form (see panel (a) in Fig. 1). It is clear that defectors dominate almost the whole parameter plane. Cooperation can only be observed when s approaches 0 or when s is large and u is very small ($u < 0.02$). As $s \to 0$, few individuals have the chance to interact before strategy updating. With most of the players having payoffs equal to zero, the natural selection is more of neutral drift. Thus the cooperation level remains around the initial value 0.5. As s increases, defectors become advantageous over cooperators in the pairwise interactions, and thus selection favors defection, leading to the dominance of defectors. When s is large, the number of chosen pairs increases and interactions become more intensive. Consequently, the interactions between individuals are no longer sparsely distributed or independent of each other, but are largely impacted by the local structures. Cooperators next to each other can form small clusters to resist the invasion of defectors, ensuring the survival of cooperators for low temptation to defect. In a word, slow selection favors cooperation in games on static structures in the sense that it supports the mechanism of spatial reciprocity [13,25], which would not be at work if the interactions are infrequent.

If we incorporate the rewiring mechanism into the system, cooperation level is greatly promoted (see panel (b) in Fig. 1). Cooperation persists for any value of s. As $s \to 0$, results resemble those observed in the no rewiring case. Otherwise, for any given value of s, the system experiences a sharp transition from the full-cooperation state to a full-defection state with increasing u. As s increases, the region of u where cooperation predominates becomes wider. This indicates that slow selection still favors cooperation in the coevolution scenario. The effect of time scale s on cooperation can be perceived in two different aspects. On one hand, similar to the above case where rewiring is absent, slow selection enforces the effectiveness of spatial structure in protecting cooperator clusters. On the other hand, slow selection has provided enough chances for the individuals to rewire their links, and thus they can avoid the bad environment in advance, before being eliminated by natural selection.