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Quantum computation with quantum-dot spin qubits inside a cavity

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ABSTRACT

Universal set of quantum gates are realized from quantum-dot spin qubits inside a cavity via two-channel Raman interactions. Individual addressing and effective switch of the cavity mediated interaction are directly possible here. This simple realization of all wanted interaction for selective qubits makes current scenario more suitable for scalable quantum computation.

Quantum computer can provide a possible alternative for certain hard problems in comparison with classical computer with the help of the principle of coherent superposition and quantum entanglement [1]. Solid state system has been generally accepted to be the most promising hardware implementation for quantum computation since it can be easily integrated into large quantum networks. Recently, with the development of fabrication and manipulation technologies in semiconductor quantum dots, quantum computation based on this system has attracted much attention. But, in a quantum dot system, decoherence is still an important and challenging issue. However, localized electron spin state has relatively long decoherence time, so it is more suitable as qubit. In addition, the realization of gate operations on arbitrary two qubits is another challenge in solid state system. In order to conquer this problem, Imamoglu and coworkers introduced the quantum dot cavity QED scheme [2] where the cavity mode can be used as a data bus for long-distance information transfer and fast coupling of arbitrary two qubits. Meanwhile, this setup can support parallel quantum logic gate operations. From then on, many schemes with quantum dots embedded in cavity have been presented [3–6].

In this Letter, we propose a scenario for realizing quantum computation with quantum dots embedded in a single-mode microcavity via a two-channel Raman interaction. Qubits are encoded on the conduction-band electron-spin states of semiconductor quantum dot. The valence-band state is used as an auxiliary state, which can be adiabatically eliminated. The decoherence time of qubits is long enough to complete indispensable gate operations. In atomic cavity QED system, the two-channel Raman interaction model has been generally acceptable as a better alternative to the single-channel one [7–11] as it can easily realize and control the needed interactions. Therefore, it is very significative to generalized the two-channel Raman interaction model to quantum dot cavity QED system for solid quantum computation. In fact, in comparison with atomic cavity QED, quantum-dot cavity QED is even more superior because quantum dots are always fixed in a cavity, thus the scale up of the solid nature system is quite straightforwardly. Meanwhile, individual addressing of quantum dot qubits, which is of great importance and challenge for scalable quantum computation, is directly possible taken into account the fact that quantum dot is generally fabricated as a mesoscopic quantum system.

We next detail our scheme. Consider N III–V semiconductor quantum dots embedded in a microcavity. All the quantum dots

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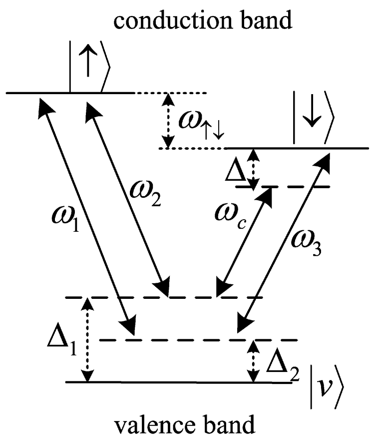

Fig. 1. The relevant energy levels of a single quantum dot. $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the spin up and down states of the conduction-band electron, respectively, and $|v\rangle$ denotes the valence-band state. ω_j $(j = 1, 2, 3)$ are the frequencies of classical laser fields and ω_c is the frequency of the cavity field.

are doped such that each quantum dot has a single conduction-band electron and a full valence band. Under the condition of quantum confinement, the conduction-band electron is always in the ground state orbital. The conduction-band electron-spin states, $|\uparrow\rangle$ and $|\downarrow\rangle$, by a uniform magnetic field are encoded as the qubit states. The relevant energy levels of every quantum dot can be treated as a three-level configuration [2,3], as shown in Fig. 1, where $\hbar\omega_\uparrow$, $\hbar\omega_\downarrow$ and $\hbar\omega_v$ are energies of the state $|\uparrow\rangle$, $|\downarrow\rangle$ and $|v\rangle$, respectively. $\omega_{\uparrow\downarrow} = \omega_\uparrow - \omega_\downarrow$ and ω_j with $j = 1, 2, 3$ are the frequencies of classical laser fields and ω_c is the frequency of the cavity field. Δ_1, Δ_2 and Δ are three detunings. Every quantum dot is excited via two Raman channels by using classical laser fields and the cavity field. One channel consists of laser fields 1 and 3, the other consists of laser field 2 and the microcavity field. Both channels are in the two-photon resonance, i.e., $\Delta_1 = \omega_\uparrow - \omega_v - \omega_2 = \omega_\downarrow - \omega_v - \omega_c - \Delta$ and $\Delta_2 = \omega_\uparrow - \omega_v - \omega_1 = \omega_\downarrow - \omega_v - \omega_3$, so we have $\omega_{\uparrow\downarrow} + \Delta = \omega_2 - \omega_c$ and $\omega_{\uparrow\downarrow} = \omega_1 - \omega_3$. The total system consists of N quantum dots, a microcavity and $3N$ classical laser fields, and the Hamiltonian of which can be described as (assuming $\hbar = 1$)

$$H = H_0 + H_{\mathrm{int}}, \tag{1a}$$

$$H_0 = \sum_{i=1}^{N} \left(\omega_\uparrow \sigma^i_{\uparrow\uparrow} + \omega_\downarrow \sigma^i_{\downarrow\downarrow} + \omega_v \sigma^i_{vv}\right) + \omega_c a^\dagger a, \tag{1b}$$

$$H_{\mathrm{int}} = \sum_{i=1}^{N} \Big[\left(\Omega_1 e^{-i\omega_1 t} + \Omega_2 e^{-i\omega_2 t}\right)\sigma^i_{\uparrow v} + \left(\Omega_3 e^{-i\omega_3 t} + ga\right)\sigma^i_{\downarrow v} + \mathrm{H.c.}\Big], \tag{1c}$$

where Ω_j with $j = 1, 2, 3$ are Rabi frequencies of classical fields, g is coupling constant, and $\sigma_{mn} = |m\rangle\langle n|$ $(m, n = \uparrow, \downarrow, v)$. In writing Eq. (1c), we have assumed that $\Omega^i_j = \Omega_j$ and $g^i = g$.

The interaction Hamiltonian (1c) can be rewritten, in the interaction picture with respect to (1b), as

$$H_I = \sum_{i=1}^{N} \Big[\Omega_2 \sigma^i_{\uparrow v} e^{i\Delta^i_1 t} + \left(\Omega_1 \sigma^i_{\uparrow v} + \Omega_3 \sigma^i_{\downarrow v}\right) e^{i\Delta^i_2 t} + ga\sigma^i_{\downarrow v} e^{i(\Delta^i_1 + \Delta^i)t} + \mathrm{H.c.}\Big]. \tag{2}$$

In the case of $\Delta_1, \Delta_2 \gg \Omega_j, g$ and $\Delta_1 - \Delta_2 \gg \{\Delta, \frac{(\Delta_1+\Delta_2)\Omega_1\Omega_2}{2\Delta_1\Delta_2}, \frac{(\Delta_1+\Delta_2)\Omega_2\Omega_3}{2\Delta_1\Delta_2}, \frac{(2\Delta_1+\Delta)\Omega_1 g}{2\Delta_1(\Delta_1+\Delta)}, \frac{(2\Delta_1+\Delta)\Omega_3 g}{2\Delta_1(\Delta_1+\Delta)}\}$, the valence-band state can be adiabatically eliminated [8]. We can then obtain an effective Hamiltonian by using rotating-wave approximation

$$H^{(1)}_e = \sum_{i=1}^{N} \left[\frac{\Omega_1\Omega_3}{\Delta^i_2}\left(\sigma^i_{\uparrow\downarrow} + \sigma^i_{\downarrow\uparrow}\right) + \frac{g\Omega_2}{2}\left(\frac{1}{\Delta^i_1} + \frac{1}{\Delta^i_1 + \Delta^i}\right) \times \left(a^\dagger \sigma^i_{\uparrow\downarrow} e^{-i\Delta^i t} + a\sigma^i_{\downarrow\uparrow} e^{i\Delta^i t}\right)\right], \tag{3}$$

where we have neglected the ac-Stark energy shift, which can be easily compensated [12] by an addition laser field dispersively coupled to an energy level outside the qubit space in real experimental implementation.

For simplification of calculation, we choose a new computational basis $|\pm\rangle^i = \frac{1}{\sqrt{2}}(|\uparrow\rangle^i \pm |\downarrow\rangle^i)$. We can rewrite the effective Hamiltonian (3) as

$$H^{(2)}_e = \sum_{i=1}^{N} \left[A\left(\frac{2S^i_z - S^i_- + S^i_+}{4} a^\dagger e^{-i\Delta^i t} + \frac{2S^i_z + S^i_- - S^i_+}{4} a e^{i\Delta^i t}\right) + BS^i_z\right], \tag{4}$$

where $A = \frac{g\Omega_2}{2}(\frac{1}{\Delta^i_1} + \frac{1}{\Delta^i_1+\Delta^i})$, $B = \frac{2\Omega_1\Omega_3}{\Delta^i_2}$, $S_+ = |+\rangle\langle-|$, $S_- = |-\rangle\langle+|$ and $S_z = \frac{1}{2}(|+\rangle\langle+| - |-\rangle\langle-|)$.

Assume that $B \gg \Delta^i, A$ and in the S_z framework $H'_0 = BS^i_z$, the Hamiltonian (4) can be reduced to

$$H_e = \sum_{i=1}^{N} \left[\frac{A}{2}\left(a^\dagger e^{-i\Delta^i t} + ae^{i\Delta^i t}\right) S^i_z\right] = \sum_{i=1}^{N} \left[\frac{A}{2}\left(a^\dagger e^{-i\Delta^i t} + ae^{i\Delta^i t}\right)\left(\sigma^i_{\uparrow\downarrow} + \sigma^i_{\downarrow\uparrow}\right)\right]. \tag{5}$$

For the implementation of quantum computation, the most important steps should be the realization of a set of universal quantum logical gates, i.e., two-qubit logic gate, controlled-not gate or controlled phase shift, and arbitrary single-qubit rotations. Here we first introduce the implementation of a controlled phase shift. We turn on three classical laser fields ω_j on quantum dots m and n, let quantum dot m interacts with quantum dot n under the condition of $\Delta^m = \Delta^n = \Delta$. Then the time evolution operator can be expressed as

$$U = e^{-i\alpha(t)(\sum_l S^l_z)^2} e^{-i\beta(t)\sum_l S^l_z a} e^{-i\gamma(t)\sum_l S^l_z a^\dagger}, \tag{6}$$

where $l = m, n$. The coefficients $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ can be calculated by solving Schrödinger equation as [13,14]

$$\beta(t) = \int_0^t \frac{A}{2} e^{i\Delta t'}\, dt' = \frac{A}{2i\Delta}\left(e^{i\Delta t} - 1\right), \tag{7a}$$

$$\gamma(t) = \int_0^t \frac{A}{2} e^{-i\Delta t'}\, dt' = \frac{-A}{2i\Delta}\left(e^{-i\Delta t} - 1\right), \tag{7b}$$

$$\alpha(t) = i\int_0^t \beta(t')\frac{A}{2} e^{-i\Delta t'}\, dt' = \frac{A^2}{4\Delta}\left[t - \frac{i}{\Delta}\left(e^{i\Delta t} - 1\right)\right]. \tag{7c}$$

Setting $\Delta t = 2\xi\pi$ $(\xi = 0, 1, \ldots)$ results in $\beta(t) = \gamma(t) = 0$ and $\alpha(t) = \frac{A^2}{4\Delta}t$. Then, the total evolution operator becomes

$$U_{m,n} = e^{-iBt(\sum_l S^l_z)} e^{-i\frac{A^2}{4\Delta}t(\sum_l S^l_z)^2}, \tag{8}$$

under which the state evolutions of $|{+}{+}\rangle_{mn}$, $|{+}{-}\rangle_{mn}$, $|{-}{+}\rangle_{mn}$ and $|{-}{-}\rangle_{mn}$ are