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Chaos synchronization of coupled neurons with gap junctions

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Abstract

Based on the asymptotic stability theory of dynamical systems and matrix theory, a general criterion of synchronization stability of N coupled neurons with symmetric configurations is established in this Letter. Especially, three types of connection styles (that is, chain, ring and global connections) are considered. As an illustration, complete synchronization of four coupled identical chaotic Chay neurons is investigated. The maximal conditional Lyapunov exponent is calculated and used to determine complete synchronization. As a result, complete synchronization of four coupled identical chaotic Chay neurons can be achieved when the coupling strength is above a critical value, which is dependent on the specific connection style. Numerical simulation is in good agreement with the theoretical analysis.

Keywords: Synchronization stability; General criterion; Asymptotic stability; Gap junction

1. Introduction

Chaos synchronization of coupled systems has been a focal topic of great interest in nonlinear science since the late 1980s. Many different synchronization phenomena have been studied including complete synchronization (CS) [1], generalized synchronization [2], phase synchronization [2–4], and lag synchronization [3]. Complete synchronization of coupled identical dynamical systems means the coincidence of the state variables with time evolution starting from different initial points in the state space. It is obvious that CS is the simplest and yet the strongest form among various synchronizations. The effect of different connection styles on CS attracts more attention in recent years. In Ref. [5], the stability of synchronization of three chaotic systems with linear coupling, can be analyzed by means of mode decomposition. This method is required to determine the sign of all eigenvalues of mode decomposition matrix based on the Routh–Hurwitz criterion. Chaos synchronization of three coupled oscillators with a ring connection was studied in [6], where unidirectional and bidirectional couplings of three identical circuits were considered. In Ref. [7], a perturbation method was used to analyze onset of synchronization of globally coupled, noisy, discrete-time systems and it seemed to be only applicable to globally coupled systems. Chaos synchronization of two mutually coupled Lorenz systems was studied by means of asymptotical stability of its linearized system and a sufficient condition was given based on the Routh–Hurwitz rule in [8]. However, in previous works, most results about synchronization stability of coupled systems were mainly based on constructing the Lyapunov function or the Routh–Hurwitz criterion. These methods required a number of rigorous calculation, especially for higher-dimensional system. This Letter presents a general criterion for synchronization stability of coupled continuous systems, which is an alternative way simpler than the previous ones. In order to test the occurrence of synchronization of a class of the coupled systems with symmetric connections, this method only depends on the calculations of the determinant and the largest nonzero eigenvalue of a sparse matrix, that is, the coupling matrix.

Neurons often behave in collective coherent motions. Physiological experiments have demonstrated the existence of synchronous firing in different areas of the brain of some animals

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like cats and monkeys [9,10]. Synchronization of coupled neurons has been suggested as a mechanism for binding globally distributed into a coherent motion, which could play a key role in intercommunications among neurons [3]. The presence or absence of synchronization can result in normal function or dysfunction of a biological system, and too well synchrony can lead to disease [3]. Hence, many research investigations are devoted to the study of synchronization of coupled neurons in recent years. Dynamical behaviour of firings in the coupled Hindmarsh–Rose (HR) neuronal systems was studied in [11, 12]. Synchronous transition of chaotic bursts in neurons was reported in [13]. A new transition style from burst synchronizations to both spike and burst synchronizations was found in coupled multi-time scale HR neural systems. In [14], the occurrence of synchronized oscillations of in-phase and anti-phase types was studied by means of two coupled FHN models with excitatory or inhibitory synaptic couplings.

It is well known that real neurons often display high nonlinearities, which has been shown in many experiments and confirmed by numerical simulation of many neuron models, including the HR and Chay neuron models [15–19]. Chaos is a universal phenomenon in nonlinear dynamics, and certain neurons, such as those in the human brain, often decode and transduce information by means of chaos. For example, olfactory cortex and hippocampus are known to exhibit chaotic dynamics [20]. Many models have been exploited to simulate chaos of neurons, and many significant results have been obtained [15–19].

Real neurons have many different connection styles, hence, they can perform different functions. In this Letter, only synchronization of N coupled identical chaotic neurons with symmetric configurations is studied. It is found that when the coupling strength exceeds a certain critical value, the N coupled chaotic neurons can go into a coherent state from non-synchronization to synchronization. This behaviour is of great importance for signal encoding and transduction in information processing of neurons in two respects: (1) single neuron may faithfully encode temporal information in the timing of successive spikes, (2) a group of neurons can respond collectively to a common synaptic current due to synchronization [21]. Meanwhile, the results of different coupling styles are compared and the mechanism of generating different dynamics is explained theoretically.

This Letter is organized as follows. Section 2 reports a general criterion of synchronization stability of N coupled chaotic neurons with symmetric configurations by means of asymptotical stability of dynamical systems and matrix theory. Three specific connections are considered based on the developed criterion in Section 3. Numerical simulation for synchronization of four coupled chaotic neurons is described in Section 4. Conclusion and discussion are presented in Section 5.

2. Dynamical analysis of synchronization of N coupled neurons with a gap junction

In neural systems, an electrical synapse is a mechanical and electrically conductive link between two abutting neurons that is formed by proteins known as gap junctions. Hence, an electrical synapse is also called a gap junction. In this kind of coupling, the synaptic current is proportional to the difference of membrane potentials between a neuron and its neighbors. Classically, electrical synapses can be thought to increase the speed and synchrony of neural activity. To understand the synchrony behaviour of coupled neurons in gap junctions with different connections, complete synchronization of N coupled identical neurons with symmetric connections is studied and a sufficient condition for synchronization is derived in this section. Dynamics of N coupled identical neurons with certain gap junctions are described by the following equations:

$$\begin{aligned} \dot{x}_{i1} &= f_1(X_i) + f(t) + g\sum_{j=1}^{N} a_{ij}x_{j1}, \\ \dot{x}_{i2} &= f_2(X_i), \\ &\vdots \\ \dot{x}_{in} &= f_n(X_i), \quad i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $X_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \mathbb{R}^n$ are state variables of the coupled ith neuron, and x_{i1} represents the membrane potential, others x_{ij} $(j \geqslant 2)$ are ionic channel dynamics. The external stimulus $f(t)$ is a time-dependent continuous function, g represents the coupling strength, and $A = (a_{ij})_{N\times N}$ is the coupling matrix.

With symmetric connections, some properties of the coupling matrix A are summarized as follows:

(1) A is a symmetric and irreducible matrix.
(2) The off-diagonal elements a_{ij} $(i \neq j)$ of A are either 1 or 0.
(3) The elements of A satisfy

$$a_{ii} = -\sum_{j=1, i\neq j}^{N} a_{ij}, \quad i = 1, 2, \ldots, N. \tag{2}$$

(4) One eigenvalue of A is zero, with multiplicity 1, and the other eigenvalues of A are strictly negative.

Given the dynamics of a single neuron and a coupling style, the stability of the synchronization state of the coupled neurons can be characterized by those nonzero eigenvalues of the coupling matrix A and the coupling strength g. In the following, some results, which determine synchronization of the coupled systems with the matrix A satisfying the above conditions, are analyzed.

Denote $F(X,t) = (f_1(X) + f(t), f_2(X), \ldots, f_n(X))$. The coupled system (1) is said to achieve synchronization if the following relation holds:

$$X_1(t) \to X_2(t) \to \cdots \to X_N(t) \to s(t), \quad \text{as } t \to +\infty, \tag{3}$$

where $s(t)$ is a solution of an isolated neuron, when the coupling strength g is zero in system (1).

Lemma 1. [22] *Consider the coupled system (1). Let*

$$0 = \lambda_1 > \lambda_2 \geqslant \lambda_3 \geqslant \cdots \geqslant \lambda_n \tag{4}$$