Physics Letters A 378 (2014) 892–898

Contents lists available at ScienceDirect

Physics Letters A

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Conditional spin counting statistics as a probe of Coulomb interaction and spin-resolved bunching

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A R T I C L E   I N F O

Article history:
Received 6 October 2013
Received in revised form 5 January 2014
Accepted 20 January 2014
Available online 27 January 2014
Communicated by R. Wu

Keywords:
Conditional spin counting statistics
Coulomb correlation
Quantum dot
Spin-resolved bunching

A B S T R A C T

Full counting statistics is a powerful tool to characterize the noise and correlations in transport through mesoscopic systems. In this work, we propose the theory of conditional spin counting statistics, i.e., the statistical fluctuations of spin-up (down) current given the observation of the spin-down (up) current. In the context of transport through a single quantum dot, it is demonstrated that a strong Coulomb interaction leads to a conditional spin counting statistics that exhibits a substantial change in comparison to that without Coulomb repulsion. It thus can be served as an effective way to probe the Coulomb interactions in mesoscopic transport systems. In case of spin polarized transport, it is further shown that the conditional spin counting statistics offers a transparent tool to reveal the spin-resolved bunching behavior.

1. Introduction

The exploration of full counting statistics (FCS) in mesoscopic systems has vital roles to play in providing penetrating insight into microscopic mechanisms in transport and temporal correlations between charge carriers which are not accessible from the conventional measurements of time-averaged current alone [1–3]. Particularly, recent advances in nanotechnology have made it possible to measure electron transport processes that take place at single-electron level [4–13]. All statistical cumulants of the number of transferred particles can now be extracted experimentally.

Theoretical study of FCS based on the scattering approach turns out to be very powerful for characterizing statistics of noninteracting electron transport through various systems, such as normal-superconductor structures [14–17], chaotic cavities [18–22], and electron entanglement detection devices [23–25]. Yet, with continued miniaturization of the system size, the involving many-particle interactions become increasingly important in mesoscopic transport [26,27]. For that purpose, a generalized quantum master equation (QME) approach has been established by Bagrets and Nazarov, with the Coulomb interactions being fully taken into account [28]. This approach has been widely employed to analyze the FCS in a variety of structures, for instance, quantum dot (QD) systems [29–35], molecules [36–40], and nanoelectromechanical resonators [41–43]. Furthermore, the QME approach was recently extended to investigate finite-frequency FCS [44,45] as well as non-Markovian dynamics [46–50].

For statistically independent tunneling events, the current fluctuations exhibit Poissonian statistics. Normally, the presence of Pauli exclusion principle, which prohibits two fermions of the same spin to be superimposed, leads to the suppression of the current noise below the Poisson value [51,52]. On the other hand, Coulomb repulsion acts as another important correlation mechanism that might enhance or inhibit noise, depending on different physical regimes concerned [53–58]. Yet, in reality it is quite difficult to distinguish the effects of Coulomb repulsion and the Pauli principle in the charge current noise. It is thus instructive to find a transparent and direct way to characterize the degree of Coulomb correlation in mesoscopic transport. For this purpose, we propose in this work the theory of conditional spin counting statistics: The statistical fluctuations of spin up (down) current given the observation of the spin down (up) current. The inspiration of this theory comes from fact that the Pauli exclusion principle only acts on fermions of the same spin, while electrons with opposite spins are only correlated via the Coulomb repulsion.

First, we consider electron transport through a single QD tunnel-coupled to two normal electrodes. Although the net current is spin unpolarized, the up and down spins are intrinsically correlated to each other via Coulomb repulsion. It is demonstrated that the Coulomb correlation gives rise to conditional spin counting statistics that exhibits a substantial change in comparison to the uncorrelated one. It thus may be utilized as an effective way to characterize the Coulomb correlation in various mesoscopic transport systems.

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0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2014.01.031

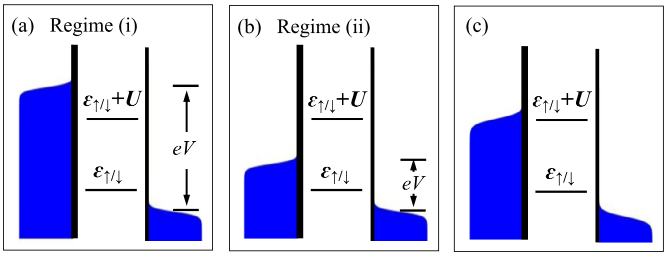

Fig. 1. Schematic setup of transport through a single QD in different bias configurations. (a) Regime (i): The bias is large enough to overcome the Coulomb interaction such that the two excitation energies ϵ_σ and $\epsilon_\sigma + U$ are within the bias window. (b) Regime (ii): A small bias is applied across the QD, and only excitation energy ϵ_σ lies in the bias window. Double occupation on the QD is prohibited. (c) The bias voltage is applied in such a way that double occupation on the QD is partially allowed. It offers an interpolation between the regimes (i) and (ii).

Second, we investigate conditional spin counting statistics for spin polarized transport by taking into account ferromagnetic electrodes. It is worthwhile to mention that the (unconditional) spin counting statistics has been studied for many years [59–62]. It was shown that spin current noise can be utilized to detect spin unit of quasiparticles [63], to sensitively probe spin decoherence in a spin battery [64], to reveal the discrete nature of the photon states for a quantum dot coupled to a cavity field [65]. In comparison with the unconditional spin counting statistics, we will show the conditional one may serve as a transparent and sensitive tool to investigate spin-resolved bunching behavior.

The rest of the paper is organized as follows. In Section 2, we describe the single QD system under different bias configurations, corresponding to different effectiveness of the Coulomb correlations. We discuss in Section 3 the charge FCS, which will be compared with the conditional spin counting statistics in sensing the Coulomb repulsion. Section 4 is devoted to the theory of conditional spin counting statistics. Its application to the single QD system is demonstrated in Section 5, with focus on its effectiveness in characterizing Coulomb correlation and spin-resolved bunching behavior. It is then followed by the conclusion in Section 6.

2. The model

We consider electron transport through a single QD with Coulomb interaction, as schematically shown in Fig. 1. The entire system is described by the Hamiltonian $H = H_{\rm B} + H_{\rm QD} + H'$, with

$$H_{\rm B} = \sum_{\alpha={\rm L,R}} \sum_{k\sigma} \epsilon_{\alpha k\sigma} c^\dagger_{\alpha k\sigma} c_{\alpha k\sigma}, \tag{1a}$$

$$H_{\rm QD} = \sum_\sigma \varepsilon_\sigma d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow, \tag{1b}$$

$$H' = \sum_{\alpha={\rm L,R}} \sum_{k\sigma} \big(t_{\alpha k\sigma} c^\dagger_{\alpha k\sigma} d_\sigma + {\rm h.c.}\big). \tag{1c}$$

Here $H_{\rm B}$ models the noninteracting electrons in the left ($\alpha = {\rm L}$) and right ($\alpha = {\rm R}$) electrodes, with $c^\dagger_{\alpha k\sigma}$ ($c_{\alpha k\sigma}$) the electron creation (annihilation) operator in the corresponding electrode. The electron distributions in the electrodes are governed by the electrochemical potentials $\mu_{\rm L}$ and $\mu_{\rm R}$, which define the voltage $eV = \mu_{\rm L} - \mu_{\rm R}$. $H_{\rm QD}$ describes the QD with one spin-degenerate energy level ε_σ and the Coulomb interaction U on the dot, where $n_\sigma = d^\dagger_\sigma d_\sigma$ is the occupation operator, with $d^\dagger_\sigma$ (d_σ) the electron annihilation (creation) operator in the QD. Electron tunneling between electrodes and QD is depicted by H'. The tunneling rate for a spin-σ electron is characterized by the intrinsic tunneling width $\Gamma_{\alpha\sigma}(\omega) = 2\pi \sum_k |t_{\alpha k\sigma}|^2 \delta(\epsilon_{\alpha k\sigma} - \omega)$. Hereafter we consider normal electrodes, i.e. $\Gamma_{\alpha\uparrow} = \Gamma_{\alpha\downarrow}$, and assume flat bands in the electrodes, which yields energy-independent couplings $\Gamma_{\alpha\sigma}$. The total tunnel-coupling strength thus is given by $\Gamma_\alpha = \Gamma_{\alpha\uparrow} + \Gamma_{\alpha\downarrow}$. Throughout this work, we set $\hbar = 1$ for the Planck constant, unless stated otherwise.

By specifying which excitation energies lie within the energy window defined by the Fermi levels $\mu_{\rm L}$ and $\mu_{\rm R}$, the following bias configurations will be considered. Regime (i): The bias is large enough to overcome the Coulomb interaction and thus the excitation energy levels ϵ_σ and $\epsilon_\sigma + U$ are within the bias window defined by chemical potentials $\mu_{\rm L}$ and $\mu_{\rm R}$, as schematically shown in Fig. 1(a). The involving states include $|0\rangle$-empty QD, $|\sigma\rangle$-single occupation by a spin-σ electron, and $|{\rm d}\rangle$-double occupation. Regime (ii): Only the single level ε_σ is within the bias window, as shown in Fig. 1(b). The charge transport is maximally correlated. The states available are $|0\rangle$-empty QD and $|\sigma\rangle$-singly occupied by a spin-σ electron. By appropriately applying the gate and bias voltages, the system can be tuned to the situation between the regimes (i) and (ii) as shown in Fig. 1(c). It allows us to analyze the effect of finite Coulomb correlation on the conditional spin counting statistics between the uncorrelated and maximally correlation cases. Our analysis is based on a second-order Born–Markov quantum master equation for $\Gamma \ll k_{\rm B}T$, where the sequential tunneling processes play the dominant role [66,67]. Higher order tunneling events, such as cotunneling, Kondo effect are thus suppressed. An approach of this type has been widely used in the literature for studying bias voltage dependent transport characteristics in various nanostructures [68], such as single [29,69,70] or double QD [71], where typical step-like transport features were revealed. In spite of the simple model considered here, we will show that it is adequate to address the essence of the conditional spin counting statistics and its effectiveness of characterizing the Coulomb correlation and investigate spin-resolved bunching characteristics.

3. Charge full counting statistics

In the single electron tunneling regime, an extra electron can inject into the QD from the left electrode, dwell in the QD for a certain amount of time before it escapes to the right electrode. This stochastic process produces intriguing signatures of the electronic conductor. To study the fluctuations involved in transport, we will utilize a Monte Carlo approach to simulate the individual electron tunneling events. We first introduce two stochastic variables ${\rm d}N_{{\rm L}\sigma}(t)$ and ${\rm d}N_{{\rm R}\sigma}(t)$ (with values either 0 or 1) to represent, respectively, the numbers of spin-σ electron injected into the QD from the left electrode and that escaped to the right electrode from the QD, during the small time interval ${\rm d}t$. One then arrives at the following conditional QME [72]

$$\begin{aligned}{\rm d}\rho^{\rm c} = {} & -{\rm i}\mathcal{L}\rho^{\rm c}(t)\,{\rm d}t - \sum_{\sigma=\uparrow,\downarrow} \big\{\Gamma_{{\rm L}\sigma}\mathcal{A}\big[d^\dagger_\sigma\big] + \Gamma_{{\rm R}\sigma}\mathcal{A}[d_\sigma] \\ & - \mathcal{P}_{{\rm L}\sigma}(t) - \mathcal{P}_{{\rm R}\sigma}(t)\big\}\rho^{\rm c}(t)\,{\rm d}t \\ & + \sum_{\sigma=\uparrow,\downarrow} {\rm d}N_{{\rm L}\sigma}\left[\frac{\Gamma_{{\rm L}\sigma}\mathcal{J}[d^\dagger_\sigma]}{\mathcal{P}_{{\rm L}\sigma}(t)} - 1\right]\rho^{\rm c}(t) \\ & + \sum_{\sigma=\uparrow,\downarrow} {\rm d}N_{{\rm R}\sigma}\left[\frac{\Gamma_{{\rm R}\sigma}\mathcal{J}[d_\sigma]}{\mathcal{P}_{{\rm R}\sigma}(t)} - 1\right]\rho^{\rm c}(t),\end{aligned} \tag{2}$$

where the involving superoperators are defined as $\mathcal{L}\rho^{\rm c} \equiv [H_{\rm QD}, \rho^{\rm c}]$, $\mathcal{J}[X]\rho^{\rm c} \equiv X\rho^{\rm c}X^\dagger$ and $\mathcal{A}[X]\rho^{\rm c} \equiv \frac{1}{2}(X^\dagger X\rho^{\rm c} + \rho^{\rm c}X^\dagger X)$. The superscript "c" attached to the reduced density matrix denotes that the quantum state is conditioned on the measurement results. For single electron tunneling events (point process), the two classical random variables satisfy