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Dust lattice wave dispersion relations in two-dimensional hexagonal crystals including the effect of dust charge polarization

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Abstract

A dusty plasma crystalline configuration with equal charge dust grains and mass is considered. Both charge and mass of each dust species are taken to be constant. Two differential equations for a two-dimensional hexagonal crystal on the basis of a Yukawa-type potential energy and a "dressed" potential energy, accounting for dust charge polarization, are derived and compared. The dispersion relation for both longitudinal and transverse wave propagation in an arbitrary direction is derived. A comparison to analytical and experimental results reported previously is carried out.

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Dusty plasma crystals represent strongly coupled dust configurations, typically occurring in plasma discharge experiments, due to the strong electrostatic interaction between massive, heavily charged, micron-sized dust particulates (dust grains) injected into the plasma [1,2]. Such dust crystals, which most often bear a two-dimensional (2D) hexagonal structure [2], support a variety of linear modes [1–5]. A theoretical treatment of longitudinal and transverse modes in Yukawa crystals including the effects of damping are investigated by Wang et al. [6]. Their theoretical predictions are in agreement with experiments. A theoretical analysis, supported by molecular dynamics simulation, of the wave dispersion relation in a 2D dust crystal in the presence of a constant magnetic field, was presented by Uchida et al. [7]. Crystal formation and dynamics have been studied in various experiments [2,8–14], where particles were essentially created by injecting artificial micro-spheres, which subsequently acquire a high (negative, usually) electron charge via inherent dynamic charging mechanisms.

Recently, Duan et al. [15] have investigated longitudinal and transverse dust grain vibrations in a 2D hexagonal lattice by considering screened Coulomb interactions (Debye–Hückel or Yukawa system) between charged dust particles, i.e., $U(r) = Q^2 \exp(-r/\lambda_D)/(4\pi\varepsilon_0 r)$, where r is the distance between two dust particles and λ_D is the Debye radius (ε_0 denotes the electric susceptibility of vacuum). Taking into account polarization due to the sheath region (near the grain surface) leads to a strong modification of the charge cloud (of opposite electric charge sign) surrounding the particles. Such a "dressing" effect in the particle interactions may even result in an attractive force between equal-sign charged dust particles [16–18].

In this Letter, we have investigated the dynamics of a two-dimensional hexagonal crystal, in search of a dispersion relation for longitudinal and transverse dust lattice waves (DLWs). The principal aspects of harmonic (linear) small-amplitude vibra-

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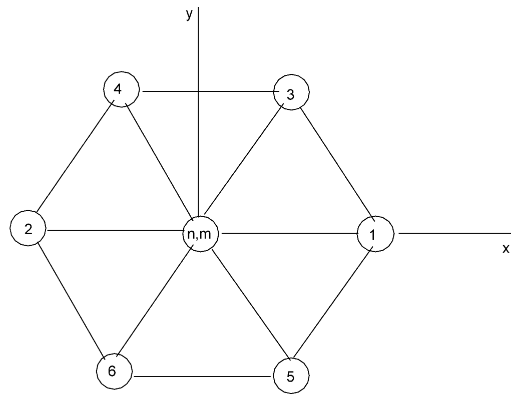

Fig. 1. The nearest neighbors around the particle (n, m) in a particle hexagonal lattice.

tional motion are investigated, by considering a "dressed" Debye-type interaction potential energy, namely, $U(r) = Q^2(1 - r/2\lambda_D)\exp(-r/\lambda_D)/(4\pi\varepsilon_0 r)$.

Let us consider a two-dimensional (2D) hexagonal crystal (assumed infinite, for simplicity) consisting of negative dust grains (of constant charge Q and mass M, for simplicity), located at equidistant sites a. For the analysis of the waves in this 2D crystalline monolayer, we use the so-called "particle string" model, allowing for two-dimensional motion, in the longitudinal (horizontal, along the x axis) and transverse (vertical) directions, the corresponding discrete ordering dust grain being denoted by indices n and m, respectively. Fig. 1 shows the nearest six particles with labels $(n+1, m)$, $(n-1, m)$, $(n+1/2, m+\sqrt{3}/2)$, $(n-1/2, m+\sqrt{3}/2)$, $(n+1/2, m-\sqrt{3}/2)$, and $(n-1/2, m-\sqrt{3}/2)$. Let the (n, m)th particle location define the origin of the plane; then the positions of the first elementary cell particles at equilibrium are $(a, 0)$, $(-a, 0)$, $(a/2, \sqrt{3}a/2)$, $(-a/2, \sqrt{3}a/2)$, $(a/2, -\sqrt{3}a/2)$ and $(-a/2, -\sqrt{3}a/2)$. However, if the particles are not at their equilibrium positions, we then define the six lengths l_1, l_2, l_3, l_4, l_5, and l_6 to represent the distances from particle (n, m) to the nearest particles, respectively,

$$l_1 = \sqrt{(a + u_{n+1,m} - u_{n,m})^2 + (v_{n+1,m} - v_{n,m})^2}, \quad (1)$$

$$l_2 = \sqrt{(a + u_{n,m} - u_{n-1,m})^2 + (v_{n,m} - v_{n-1,m})^2}, \quad (2)$$

$$l_3 = \sqrt{(a/2 + u_{n+1/2,m+\sqrt{3}/2} - u_{n,m})^2 + \left(\sqrt{3}a/2 + v_{n+1/2,m+\sqrt{3}/2} - v_{n,m}\right)^2}, \quad (3)$$

$$l_4 = \sqrt{(a/2 - u_{n-1/2,m+\sqrt{3}/2} + u_{n,m})^2 + \left(\sqrt{3}a/2 + v_{n-1/2,m+\sqrt{3}/2} - v_{n,m}\right)^2}, \quad (4)$$

$$l_5 = \sqrt{(a/2 + u_{n+1/2,m-\sqrt{3}/2} - u_{n,m})^2 + \left(\sqrt{3}a/2 - v_{n+1/2,m-\sqrt{3}/2} + v_{n,m}\right)^2}, \quad (5)$$

$$l_6 = \sqrt{(a/2 - u_{n-1/2,m-\sqrt{3}/2} + u_{n,m})^2 + \left(\sqrt{3}a/2 - v_{n-1/2,m-\sqrt{3}/2} + v_{n,m}\right)^2}, \quad (6)$$

where u and v are the particle displacements from their equilibrium positions in the x and y directions, respectively. The electrostatic binary interaction force $F(r)$ exerted onto two dust grains situated at a distance r is derived from a potential function $U(r)$, viz. $F(r) = -\partial U(r)/\partial r$. We may expand the potential energy around equilibrium at $r = a$, viz.

$$U(r) = U(a) + (r-a)\frac{\partial U}{\partial r}\bigg|_{r=a} + \frac{1}{2}(r-a)^2\frac{\partial^2 U}{\partial r^2}\bigg|_{r=a} + \cdots. \quad (7)$$

By defining the "spring" constant $G = (\partial^2 U/\partial r^2)|_{r=a}$, and setting the potential energy at equilibrium to zero, we have

$$U(r) \cong \frac{1}{2}G(r-a)^2. \quad (8)$$

We have calculated G, for a Yukawa system $U_1(r)$ on one hand, and for a "dressed" potential energy $U_2(r)$, on the other; the corresponding expressions read

$$G_1 = \frac{2Q^2}{4\pi\varepsilon_0\lambda_D^3}\frac{1+\kappa+\kappa^2/2}{\kappa^3}e^{-\kappa}, \quad (9)$$

$$G_2 = \frac{2Q^2}{4\pi\varepsilon_0\lambda_D^3}\frac{1+\kappa+\kappa^2/2-\kappa^3/4}{\kappa^3}e^{-\kappa}, \quad (10)$$

respectively, where we have defined the (dimensionless) lattice parameter $\kappa = a/\lambda_D$. Fig. 2 shows the harmonic potential energy,