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Macroscopic entanglement on a hybrid quantum circuit

Xiao-Ling He [a], Sheng Li [b], Yu-Shen Liu [c], Thomas F. George [d,*]

[a] School of Science, Zhejiang University of Science and Technology, Hangzhou, Zhejiang 310023, China
[b] Department of Physics, Zhejiang Normal University, Jinhua, Zhejiang 321004, China
[c] Department of Physics, Changshu Institute of Technology, Jiangsu 215500, China
[d] Office of the Chancellor and Center for Nanoscience, Departments of Chemistry & Biochemistry and Physics & Astronomy, University of Missouri-St. Louis, St. Louis, MO 63121, USA

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ABSTRACT

A scheme is proposed to deterministically create maximal entanglement between hybrid artificial atoms: superconducting charge and flux qubits. By tuning the circuit, the two qubits are dynamically decoupled and entanglement can be long-lived. This provides a new version of the Einstein–Podolsdy–Rosen (EPR) situation where the components of a macroscopic EPR pair are in opposite regimes.

1. Introduction

Entanglement is one of the bases of quantum computation which, generally, is realized through the controllable evolution of entangled quantum states. Among quantum computer architectures, superconducting qubits, such as the charge qubit [1] and flux qubit [2], are widely regarded as candidates for quantum computation. There has been much interest in the manipulation of these qubits, or artificial atoms [3,4]. Given that superconducting circuits contain as many as 10^{11} atoms, a meaningful step is to engineer the circuits to deterministically exhibit quantum correlations between macroscopically spatially separated "atoms". These on-chip atoms provide a realistic laboratory to test the validity of the principles of quantum mechanics at a macroscopic level.

In a study a decade ago, experiments using entangled photon pairs have been implemented [5], but these could not be manipulated for the purpose of achieving overall entanglement, due to spontaneous processes. Fortunately, two years later, remarkable progress on the trapping of ions led successfully to entanglement [6]. Because the strong ion–ion coupling results in a small spatial separation between the ions, addressing the ions individually becomes a new problem. Recent developments in the manipulation of coupled Josephson systems [7,8] allows the design of exquisite experiments to study quantum correlations between two macroscopic degrees of freedom in a nano-electronic circuit.

To realize the coupling and entanglement between superconducting qubits, several theoretical methods have been proposed: (1) switching on/off via the magnetic flux through a superconducting quantum interference device (SQUID) loop [9–11] by changing the amplitude of the DC signal; (2) adiabatically tuning the qubit frequency [8,12–15] or adding auxillary subcircuits [16,17]; and (3) controlling the coupling by an external variable-frequency magnetic field at the combination frequencies of the two qubits [18]. Up to now, almost any scheme for coupling superconducting qubits supposes only one kind of artificial atom. As is well known, the charge qubit has a more flexible control style than the flux qubit, but has a shorter coherence time than that of the flux qubit [9]. In order to take the advantage of the different kinds of qubits and communicate among them, it is necessary to entangle them. However, due to the uncertainty principle, the number of Cooper pairs on the island of the charge qubit is well fixed and the phase strongly fluctuate, while the phase of the flux qubit is also well defined. Thus, how to couple the two qubits working in opposite regimes and being restrained by the uncertainty principle becomes the key challenge in the scheme.

Successfully addressing this challenge is the purpose of this Letter. Here, we propose a controllable coupling scheme to maximally entangle a charge qubit and flux qubit via a large Josephson junction (JJ) [19–21], where the coupling is strong enough to quickly achieve maximal entanglement. By tuning the elements of the circuit, the two qubits are dynamically decoupled to form long-lived entanglement, which makes it convenient to flexibly engineer the quantum state of the circuit. Furthermore, we can realize a new version of the EPR situation where the components of an EPR pair are in opposite regimes, illustrating macroscopic quantum behav-

* Corresponding author.
E-mail address: tfgeorge@umsl.edu (T.F. George).

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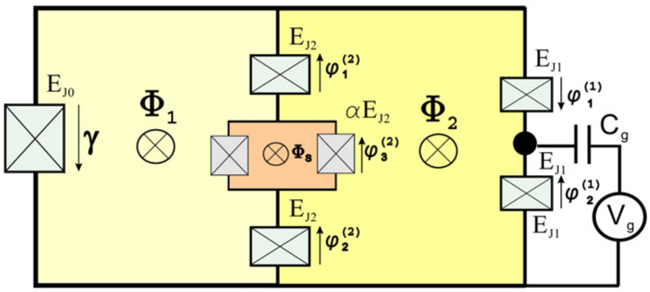

Fig. 1. (Color online.) Two artificial two-level atoms (charge qubit on the right side and flux qubit in the middle) coupled via a large JJ. The relationship of the Josephson energy of the JJs in the circuit is $E_{J0} \gg E_{J2} \gg E_{J1}$. The SQUID (two small JJs) of the flux qubit provides an adjustable effective Josephson energy. The evolution of quantum states of the two qubits and the coupling are manipulated by the external control parameters: fluxes Φ_i and Φ_s and biased voltage V_g.

ior. Finally, we briefly introduce a measurement process for the generated EPR pairs.

2. Coupling model

2.1. Hamiltonian of the superconducting circuit

In the schematic diagram shown in Fig. 1, the two JJs in the right division work in the charge regime and are associated with the charge qubit [1]. The JJs in the middle division work in the phase regime and are associated with the flux qubit [2]. Compared with the other above JJs, the JJ in the left division has small charge energy and large Josephson energy [20]. Due to the non-dissipative phase correlation in a superconducting circuit, the middle and right divisions can be fabricated at a distance apart up to a few micrometers [7].

The right division consists of two identical JJs with Josephson energy E_{J1} and capacitance C_1. The superconducting island with excess charge $2ne$ is biased by a voltage V_g via a gate capacitance C_g. The Hamiltonian of the right division contains the charge energy and Josephson energy,

$$H_1 = E_{C1}\left(n - \frac{C_g V_g}{2e}\right)^2 - E_{J1}\left(\cos\varphi_1^{(1)} + \cos\varphi_2^{(1)}\right), \tag{1}$$

where $E_{C1} = 2e^2/C_1$. The number n of excess Cooper pairs on the superconducting island is canonically conjugate to the average phase drop of the two JJs $\varphi_A^{(1)} = (\varphi_1^{(1)} + \varphi_2^{(1)})/2$. The JJs of the charge qubit are correlated to the large JJ by the gauge invariants, which are restricted by the flux quantization around a loop containing the phase drops of the JJs,

$$\varphi_1^{(1)} - \varphi_2^{(1)} - \gamma + 2\pi f_1 = 0, \tag{2}$$

where $f_1 = (\Phi_1 + \Phi_2 + \Phi_s)/\Phi_0$ is the reduced external flux and Φ_0 is the magnetic flux quantum. Using the above flux quantization restriction, Eq. (1) is rewritten as

$$H_1 = E_{C1}\left(n - \frac{C_g V_g}{2e}\right)^2 - 2E_{J1}\cos\left(f_1\pi - \frac{\gamma}{2}\right)\cos\varphi_A^{(1)}. \tag{3}$$

The middle division of the circuit contains two identical JJs (C_2, E_{J2}) and a symmetric SQUID with two smaller JJs $(\beta C_2, \beta E_{J2})$. The phase difference of the SQUID is $\varphi_3^{(2)}$. The SQUID provides an adjustable effective Josephson energy αE_{J2} with

$$\alpha = 2\beta\cos(f_s\pi), \tag{4}$$

where $f_s = \Phi_s/\Phi_0$ is the reduced external flux. Experimentally, the flux qubit parameter α lies in the range 0.5–0.8, where its actual value is determined by β and f_s. The Hamiltonian of the middle division consists of the charge energy (effective momental term) and Josephson energy (effective potential term) [22],

$$H_2 = \frac{P_P^2}{2M_P} + \frac{P_Q^2}{2M_Q} + U(\varphi_Q, \varphi_P), \tag{5}$$

with effective masses $M_Q = 2(\Phi_0/2\pi)^2 C_2$ and $M_P = (1+4\beta)M_Q$. The effective potential is

$$U(\varphi_Q, \varphi_P) = E_{J2}(2 - 2\cos\varphi_Q\cos\varphi_P) + 2\beta E_{J2} - 2\beta E_{J2}\cos(f_s\pi)\cos(2\varphi_P + \gamma - 2\pi f_2), \tag{6}$$

with the reduced external flux $f_2 = \Phi_1/\Phi_0$. The redefined phases

$$\varphi_P = \left(\varphi_1^{(2)} + \varphi_2^{(2)}\right)/2, \tag{7}$$

$$\varphi_Q = \left(\varphi_1^{(2)} - \varphi_2^{(2)}\right)/2 \tag{8}$$

correspond to the effective momenta $P_Q = M_Q(\partial\varphi_Q/\partial t)$ and $P_P = M_P(\partial\varphi_P/\partial t)$, respectively. The JJs of the flux qubit are correlated to the large JJ by the gauge invariants, which are also restricted by the flux quantization

$$\sum_{i=1}^{3}\varphi_i^{(2)} + \gamma - 2\pi f_2 = 0. \tag{9}$$

Since the charge energy of the large JJ is sufficiently small, we can neglect that energy, and the Hamiltonian of large JJ can be reduced to

$$H_M = -E_{J0}\cos\gamma. \tag{10}$$

Thus the large JJ behaves as a classical JJ.

2.2. Phase relationships

In order to achieve an effective coupling for the reduced charge and flux qubits, we should divide the Hamiltonian of the circuit into the Hamiltonians of the two independent qubits and the interaction between them. Therefore, we should replace γ in Eqs. (3) and (5) with the phase drops of qubits. In terms of Kirchhoff's current law, the supercurrents of the right and middle divisions contribute to that of the left division,

$$I_1 + I_2 = I_0, \tag{11}$$

where the supercurrent of the right division is

$$I_1 = I_c^{(1)}\sin\varphi_1^{(1)} - I_c^{(1)}\sin\varphi_2^{(1)} = 2I_c^{(1)}\sin\left(\frac{\gamma}{2} - f_1\pi\right)\cos\varphi_A^{(1)}, \tag{12}$$

with the critical current $I_c^{(1)} = 2\pi E_{J1}/\Phi_0$. The supercurrent of the middle division is

$$I_2 = I_{3c}^{(2)}\sin\varphi_3^{(2)} = I_{3c}^{(2)}\sin\left(-\varphi_1^{(2)} - \varphi_2^{(2)} - \gamma + 2\pi f_2\right), \tag{13}$$

with the critical current of the effective JJ of the SQUID as $I_{3c}^{(2)} = 2\pi\alpha E_{J2}/\Phi_0$. The supercurrent of the left division is

$$I_0 = I_c^{(0)}\sin\gamma, \tag{14}$$

with $I_c^{(0)} = 2\pi E_{J0}/\Phi_0$. Combining Eqs. (12)–(14), Eq. (11) is rewritten as

$$\sin\gamma = 2\eta_1\sin\left(\frac{\gamma}{2} - f_1\pi\right)\cos\varphi_A^{(1)} - \eta_2\sin\left(\varphi_1^{(2)} + \varphi_2^{(2)} + \gamma - 2\pi f_2\right), \tag{15}$$