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Complicated basins and the phenomenon of amplitude death in coupled hidden attractors

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ABSTRACT

Understanding hidden attractors, whose basins of attraction do not contain the neighborhood of equilibrium of the system, are important in many physical applications. We observe riddled-like complicated basins of coexisting hidden attractors both in coupled and uncoupled systems. Amplitude death is observed in coupled hidden attractors with no fixed point using nonlinear interaction. A new route to amplitude death is observed in time-delay coupled hidden attractors. Numerical results are presented for systems with no or one stable fixed point. The applications are highlighted.

1. Introduction

Attractors are termed as self-excited attractors if their basins intersect with the neighborhood of equilibria present in the system. Such attractors in various systems, e.g. Lorenz, Rossler, Chua, etc., have been studied in detail [1–3]. Very recently, a new type of attractors called hidden attractors, that don't intersect with the neighborhood of any equilibrium, have been reported [4–6]. Due to absence of unstable equilibrium in its neighborhood these type of attractors are less traceable. Therefore, it is also difficult to understand their characteristic behavior [4–11]. Even to locate the attractors in a given system requires proper search methods [8–11]. In recent years various attempts have been made to understand such attractors. Various physical as well as mathematical models have been explored with possibility of finding hidden oscillating attractors in systems having no [12–15], one [12,16], and more [17–20] stable fixed point. Understanding the properties of such attractors are important as they are observed in various systems e.g. Chua, Electrical Machines, drilling system, etc. [4,5,7,12,19]

Systems with coexisting attractors have a complex dynamical behavior due to the extreme sensitivity towards initial conditions, system parameter, and noise [1,21,22]. The presence of coexisting attractors in a system creates a dilemma in deciding the final asymptotic state of the system. In many practical situations, particularly from an engineering point of view, understanding the structures of the basins of such attractors are essential [23,24]. In this paper we study coexisting hidden attractors which have riddled-like complicated basins.

Natural systems are rarely isolated, and hence the interaction between such systems have been extensively studied for self-excited systems, both from theoretical and experimental points of view. Several interesting new phenomena have been observed in such interacting systems [3,25,26]. One such phenomena, amplitude death (AD) [27], is important because it can also occur in coupled nonlinear oscillators. It occurs when interaction causes the fixed points to become stable and attracting. Since no fixed point or a set of stable fixed points exist in systems with hidden attractors, understanding the nature and consequences of coupling in such systems is equally important. For a hidden attractor having no-stable point, AD can be induced only by creating new fixed points in the coupled system. However, for the case of hidden attractors having stable fixed points, AD, can be observed using appropriate interactions. In this paper we show that AD can be achieved using nonlinear interaction in systems having no-fixed points while in systems having stable fixed points this can be achieved using time-delay interaction. A new route to AD is observed for the case of stable fixed point systems that is very different from existing routes to AD in self-excited attractors [27].

This paper is organized as follows. In Section 2 we study individual systems with no or one stable fixed point. The riddled-like basin for coexisting hidden attractors are presented in this section. The phenomena of AD due to nonlinear and time-delayed interactions in coupled hidden attractors are also demonstrated. In Section 3 summary, conclusions are presented.

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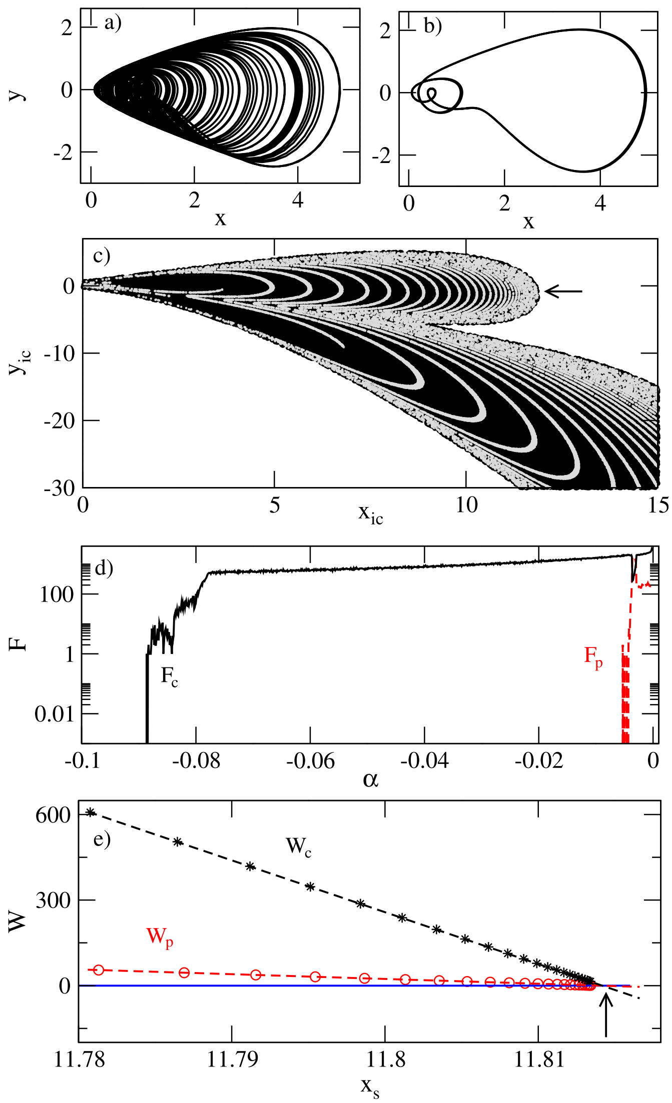

Fig. 1. (Color online.) The trajectories of (a) chaotic and (b) periodic attractors, and (c) the corresponding basins (chaotic – black and periodic – grey colors) at $\alpha = -0.001$. (d) The fractions of initial conditions which go to chaotic (F_c, solid black line) and periodic (F_p, dashed red line) attractors as a function of parameter α. (e) The width of the strips of period and chaotic basins as a function starting positions x_s of the strips, along the marked arrow in (c).

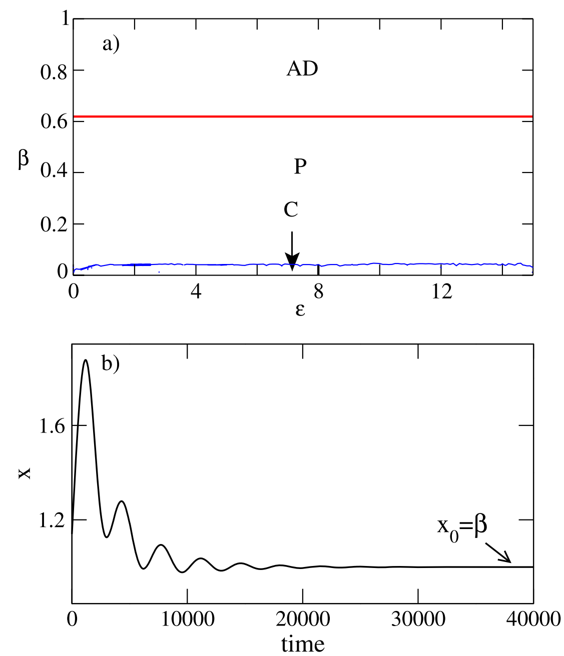

Fig. 2. (Color online.) (a) The schematic phase diagram in parameters ϵ and β of Eq. (2). (b) The transient trajectory for targeted fixed point solution in AD region of (a).

2. Results and discussions

In this section, we study a particular class of systems having either no or one fixed point. However, similar results are observed in systems having two or more stable fixed points as well [28].

2.1. System with no fixed point

We first consider a system that has no fixed point and provides hidden attractors [12],

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -y + 3y^2 - x^2 - xz + \alpha. \end{aligned} \tag{1}$$

This system doesn't have any fixed point for parameter $\alpha < 0$ [12]. This system has been studied earlier where the existence of a single chaotic attractor has been demonstrated [12]. Such a typical chaotic attractor is shown in Fig. 1(a) for $\alpha = -0.001$. Apart from this attractor, a new periodic attractor is also observed at the same parameter as shown in Fig. 1(b). Because there is no fixed point in this system at this parameter, these coexisting attractors are termed as hidden ones. Since these coexisting attractors depend on initial conditions, we explore their corresponding basins. These are shown in Fig. 1(c). The dark and brown strips (within dark region) correspond to the chaotic and periodic motions respectively. The blank (white) regions correspond to the initial conditions which don't have bound solutions, i.e., system goes to infinity. These basins are generated from 10^6 random initial conditions in the range of $x_{ic} \in (0, 15)$, $y_{ic} \in (-30, 40)$ and $z_{ic} = 0$. The basins of chaotic and periodic motions are determined from the largest Lyapunov exponents (LE) [29] i.e., for LE > 0.05 corresponds to the chaotic attractor (criteria for systems having Perron effect – see Refs. [30,31]) otherwise motion is considered as periodic one. Apart from these coexisting attractors no other attractor was found in this range of the initial conditions.

In order to show how the basins of these attractors change as a function of the parameter α, shown in Fig. 1(d) are the plot of fractions (F) of initial conditions which go to either chaotic (solid line) or periodic (dashed line) attractors. Here the fractions are calculated by considering 10^4 random initial conditions (in the range used for Fig. 1(b)) and averaged over the number of initial conditions which go to the oscillatory motion (chaotic or periodic). This figure shows that there is no bound solution for $\alpha \lesssim -0.09$. However as α is increased, the chaotic attractor appears. As α is further increased, for $\alpha \gtrsim -0.005$, both the chaotic as well as the periodic attractors coexist (Fig. 1(b)). From these analysis it is concluded that the systems with hidden attractors can have different types of oscillating dynamics as well as complicated basins. Therefore, such systems need to be extensively studied for a complete understanding of its characteristic behavior.

As we see in Fig. 1(c), the strips corresponding to the periodic or chaotic regime are becoming thinner and thinner near the boundary of oscillatory and unbounded solutions. In order to see how the widths (W) of these strips are changing, we consider the initial conditions on a fixed line at $y_{ic} = 0$ (indicated by arrow in Fig. 1(c)). We calculate the width of the each strip which