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Nonlocal Andreev reflection in a three-terminal Aharonov–Bohm interferometer

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Abstract

We theoretically report a nonlocal Andreev reflection in an Aharonov–Bohm interferometer, which is a three-terminal normal metal/superconductor (NS) mesoscopic hybrid system. It is found that this nonlocal Andreev reflection is sensitive to the systematic parameters, such as the bias voltages, the quantum dot levels, and the external magnetic flux. If we set the chemical potential of one normal metal lead equal to zero, the electronic current in the lead results from two competing processes: the quasiparticle transmission and nonlocal Andreev reflection. The appearance of zero electronic current signals unambiguously the existence of this nonlocal Andreev reflection.

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There is growing interest in the mesoscopic normal metal/superconductor hybrid systems [1–6] in the last decade, where the so-called Andreev reflection [7] happens at the NS interface due to the appearance of a new energy scale: the superconducting energy gap Δ. This phenomenon indicates that an incoming electron from normal side is reflected as a hole, thereby transferring a cooper pair into the superconducting condensate. When there are two normal metal leads in NS hybrid systems, it is possible for an electron from a normal metal lead to be reflected as a hole into another spatially separated lead. This process is the well-known nonlocal Andreev reflection [8–15]. Since this nonlocal Andreev reflection process is equivalent to injecting two spin-entangled electrons which form the singlet state of a Cooper pair in two different normal leads, the study of electron transport in the multiterminal NS hybrid systems may lead to the realization of solid-state entanglers [9] that are of important to quantum information processing. In addition, with the recent developments in the micro-fabrication technology, many artificial structures in mesoscopic scale can be fabricated. Electrons move in the mesoscopic regime can avoid the inelastic scattering and keep the phase coherence. This kind of phase coherent electron transport has been widely investigated by the solid-state Aharonov–Bohm interferometer [16–27].

In this work we will address both the nonlocal Andreev reflection process and the phase coherent transport in a three-terminal Aharonov–Bohm interferometer. The system that we consider is illustrated in Fig. 1. Two normal metal leads and a superconducting lead are coupled to their respective quantum dots to form an Aharonov–Bohm ring. The model Hamiltonian of this system reads

$$H = \sum_{i=1,2} H_{Ni} + H_S + H_{\mathrm{dot}} + H_c. \tag{1}$$

Here $H_{Ni} = \sum_{k\sigma} (\varepsilon_{ik} + qV_i) c^\dagger_{ik\sigma} c_{ik\sigma}$ is the Hamiltonian of the ith normal metal lead. $H_S = \sum_q [\sum_\sigma \varepsilon_q c^\dagger_{q\sigma} c_{q\sigma} + \Delta c^\dagger_{q\uparrow} c^\dagger_{-q\downarrow} + \Delta c_{-q\downarrow} c_{q\uparrow}]$ is the Hamiltonian of the superconducting lead with

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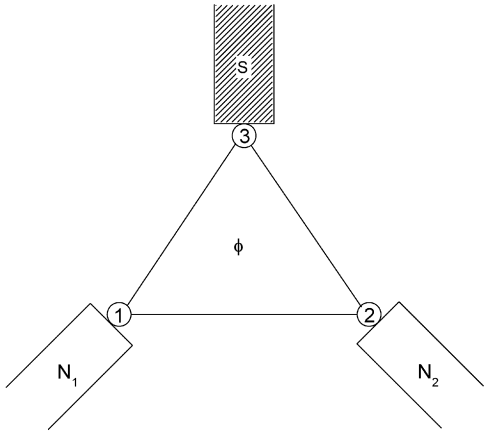

Fig. 1. Schematic diagram of our system.

Δ the superconductor energy gap. Note that the bias voltages V_i are applied in the normal metal leads and zero chemical potential is kept in the superconducting lead. What we are only interested is the Andreev reflection process, so we assume the bias voltage V_i are lesser than the superconducting energy gap Δ. $H_{\mathrm{dot}} = \sum_\sigma [\sum_{j=1,2,3} \varepsilon_j d_{j\sigma}^\dagger d_{j\sigma} + (t_{12} e^{\frac{i\varphi}{3}} d_{1\sigma}^\dagger d_{2\sigma} + t_{23} e^{\frac{i\varphi}{3}} d_{2\sigma}^\dagger d_{3\sigma} + t_{31} e^{\frac{i\varphi}{3}} d_{3\sigma}^\dagger d_{1\sigma} + \text{c.c.})]$ is the Hamiltonian for the quantum dots, and the discrete bare energy level ε_j are well controlled by the corresponding gate voltages. H_c is the Hamiltonian standing for the couplings between the quantum dots and three leads, which is given by

$$H_c = \sum_{k\sigma, i=1,2} T_{ik}^N c_{ik\sigma}^\dagger d_{i\sigma} + \sum_{q\sigma} T_q^S c_{q\sigma}^\dagger d_{3\sigma} + \text{h.c.} \tag{2}$$

For simplicity, we have assumed that the hopping matrix elements are independent of the spin index.

We now calculate the electronic current passing through the second normal metal lead, which is defined as

$$I_{N2} = q\left\langle \frac{d\hat{N}_{N2}}{dt} \right\rangle, \tag{3}$$

with $\hat{N}_{N2} = \sum_{k\sigma} c_{2k\sigma}^\dagger c_{2k\sigma}$. From Heisenberg equation of motion, the electronic current can be rewritten as ($\hbar = 1$)

$$\begin{aligned} I_{N2} = -2q\Gamma_{N2} \int \frac{dE}{2\pi} \big[& \Gamma_{N1} |G_{31}^r|^2 (f_2^- - f_1^-) \\ & + \Gamma_{N2} |G_{34}^r|^2 (f_2^- - f_2^+) \\ & + \Gamma_{N1} |G_{32}^r|^2 (f_2^- - f_1^+) \big], \end{aligned} \tag{4}$$

where $f_i^\pm(E) = 1/(e^{\beta(E \pm qV_i)} + 1)$ are the well-known Fermi distribution functions, $\Gamma_{Ni} = 2\pi \sum_k |T_{ik}^N|^2 \delta(E - \varepsilon_{ik})$ are the linewidth functions which describe the coupling strength between the ith normal metal lead and the corresponding quantum dot. G^r are the Fourier transformation of the retarded Green's functions for the quantum dots in 6×6 Nambu representation [4,27]

$$G^r(t, t') = -i\theta(t - t') \langle \{ \Psi(t), \Psi^\dagger(t') \} \rangle, \tag{5}$$

with row vector $\Psi^\dagger = (d_{1\uparrow}^\dagger, d_{1\downarrow}, d_{2\uparrow}^\dagger, d_{2\downarrow}, d_{3\uparrow}^\dagger, d_{3\downarrow})$. In order to obtain the electronic current, one must know expressions of the retarded Green's functions for the quantum dots. The most convenient way to calculate the retarded Green's function is the Dyson equation, which is given by

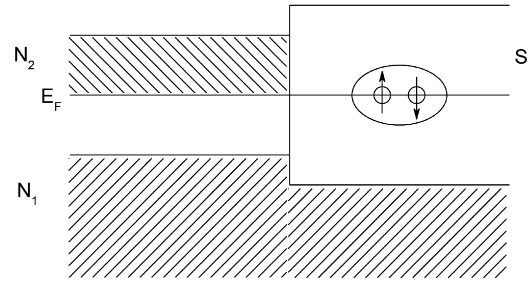

Fig. 2. The vanishing nonlocal Andreev reflection when the biases satisfy $V_1 = -V_2$.

$$G^r(E) = \frac{1}{G_0^r(E)^{-1} - \Sigma^r(E)}, \tag{6}$$

with

$$\begin{aligned} & G_0^r(E)^{-1} \\ & = \begin{pmatrix} E - \varepsilon_1 & 0 & -t_{12} e^{\frac{i\varphi}{3}} & 0 & -t_{13} e^{\frac{-i\varphi}{3}} & 0 \\ 0 & E + \varepsilon_1 & 0 & t_{12} e^{\frac{-i\varphi}{3}} & 0 & t_{13} e^{\frac{i\varphi}{3}} \\ -t_{12} e^{\frac{-i\varphi}{3}} & 0 & E - \varepsilon_2 & 0 & -t_{23} e^{\frac{i\varphi}{3}} & 0 \\ 0 & t_{12} e^{\frac{i\varphi}{3}} & 0 & E + \varepsilon_2 & 0 & t_{23} e^{\frac{-i\varphi}{3}} \\ -t_{13} e^{\frac{i\varphi}{3}} & 0 & -t_{23} e^{\frac{-i\varphi}{3}} & 0 & E - \varepsilon_3 & 0 \\ 0 & t_{13} e^{\frac{-i\varphi}{3}} & 0 & t_{23} e^{\frac{i\varphi}{3}} & 0 & E + \varepsilon_3 \end{pmatrix} \end{aligned} \tag{7}$$

and

$$\Sigma^r(E) = -\frac{i}{2} \begin{pmatrix} \Gamma_{N1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \Gamma_{N1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_{N2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Gamma_{N2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_S \beta_1 & \Gamma_S \beta_2 \\ 0 & 0 & 0 & 0 & \Gamma_S \beta_2 & \Gamma_S \beta_1 \end{pmatrix}, \tag{8}$$

where Γ_S is the linewidth function of electrons in the superconducting lead, which has the similar expression of Γ_{Ni}. Other symbols $\beta_1 \equiv \zeta(E)E/\sqrt{E^2 - \Delta^2}$, $\beta_2 \equiv \zeta(E)\Delta/\sqrt{E^2 - \Delta^2}$ with $\zeta(E) = 1$ for $E > -\Delta$, otherwise $\zeta(E) = -1$. Eq. (4) is one of the central results of this Letter, which is valid for any temperatures. The physical meanings of Eq. (4) are as follows: The first term corresponds to the quasiparticle current due to the applied bias ($V_2 - V_1$) between the normal metal leads 2 and 1. The second term is the usual Andreev reflection describing an electron in lead 2 is reflected as a hole into the same lead. The most interesting nonlocal Andreev process can be seen in the third term, and an electron in the normal metal lead 2 is reflected as a hole into the normal metal lead 1.

Before our numerical calculations, we first note that the nonlocal Andreev reflection vanishes when the bias voltages satisfy $V_1 = -V_2$. This phenomenon can be well understood from Fig. 2: An incident electron with the energy from 0 to $+V_2$ in normal metal lead 2 cannot reflected as a hole into the normal metal lead 1 since the hole states with energy from 0 to