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Optimal shape design of iron pole section of electromagnet

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Abstract

In this Letter, the classical problem of determining the optimal shape of the pole of an electromagnet is considered. In order to determine the optimal shape, we have extended a measure theory-based method. The problem of finding the optimal shape is reduced to one consisting of minimizing a linear form over a set of positive measures. To do so, an embedding procedure is applied. The resulting problem can be approximated by a finite-dimensional linear programming problem. The solution of the problem is used to construct a nearly optimal shape.

MSC: 62K05; 28A33; 49J20; 90C05

Keywords: Optimal shape design; Electromagnet; Optimal control; Measure space; Linear programming

1. Introduction

Optimization of the shape of an electromagnet is one of the classical problems in shape optimization. I.D. Lukáš, in Chapter 7 of [1], has found optimal shapes of two electromagnets in order to minimize inhomogeneities of the magnetic field in a certain area, where he discusses the speedup of the used adjoint method comparing to the numerical differentiation, and the speedup of the multilevel approach with respect to the standard approach. The finite element method (FEM) and the boundary element method (BEM) are applied in shape optimization of electromagnets. One can see for example the applications of FEM in Pironneau [2] and Hiptmair [3], and for BEM, Kaltenbacher et al. [4], and C.S. Koh et al. [5], where the sensitivity analysis for two-dimensional electromagnetic systems is described by using implicit differentiation and direct boundary element methods. Peichl et al. [6] analyze the relationship between gradients for finite-dimensional optimization problems and the derivative of the infinite-dimensional cost in the optimal shape of electromagnets.

In this Letter, we transform the problem of optimal design of an electromagnet shape to an optimal control problem accompanied with a boundary value partial differential equation. In order to solve the optimal control problem, we have extended a measure theory-based approach. Developed by Rubio [7], measure theory is an effective method for solving optimal control problems. The advantages of the proposed method lies in the fact that the method is not iterative, it is self-starting, and it does not need to solve corresponding boundary value problems. Because of these features, this method has been extended to solve a variety of control problems. We mention only [8–13].

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2. The shape optimization problem

Consider the cross section of a device shape depicted in Fig. 1. The electromagnet consists of an iron core ω_1 and a coil which penetrates the cross section plane at W_1 and W_2, respectively. A current J flows in the coil, pointing outward on W_1 and inward on W_2. The current density is supposed to be constant over the cross section of the coil. The material-dependent magnetic reluctivities are given by constants ν_1 in the iron region, by ν_2 in copper and air.

The planar magnetic field $\mathbf{B} = (B_1, B_2, 0)$ is determined by $\mathbf{B} = \operatorname{curl} \mathbf{A}$, where $\mathbf{A} = (0, 0, A)$. The electromagnetic potential A satisfies the equation

$$-\operatorname{div}\bigl(\nu(x_1, x_2)\nabla A\bigr) = J(x_1, x_2), \quad (x_1, x_2) \in \omega, \tag{1}$$

where

$$\nu(x_1, x_2) = \begin{cases} \nu_1, & \text{on } \omega_1 \\ \nu_2, & \text{elsewhere} \end{cases}$$

and

$$J(x_1, x_2) = \begin{cases} j, & \text{on } W_1 \\ -j, & \text{on } W_2 \\ 0, & \text{elsewhere} \end{cases}$$

while j is the current density. A physically reasonable boundary condition for A is given by

$$A_{\partial\omega} = 0, \tag{2}$$

where the domain ω is chosen such that $\omega_1 \subset \omega$ and $\partial\omega$ is sufficiently far away from the source of the magnetostatic field (see [6]). Set $\omega_2 = \omega - \overline{\omega_1}$. We are interested in designing the pole such that the electromagnet field is as close as possible to a desired vector $u_d = (u_{d_1}, u_{d_2})^T$ in the given area of D. A cost functional which realizes this objective is given in [2] as

$$\mathcal{J} = \frac{1}{2} \int_D \|\nabla A - u_d\|_2^2 \, dx_1 \, dx_2. \tag{3}$$

Our goal is to find the optimal shape of the iron pole which minimizes the cost functional (3). We assume that only a part of the boundary $\partial\omega_1$ of the iron core is variable, and denote this part by Θ, where now Θ is parameterized as (see Fig. 1),

$$\Theta = \bigl\{\bigl(x_1, \delta(x_1)\bigr): \ x_1 \in [o, p]; \ \delta \in U^{ad}\bigr\}. \tag{4}$$

The family of admissible shape functions δ are characterized by

$$U^{ad} = \bigl\{f \in W_0^{2\infty}(o, p): \ t \leqslant f(x_1) \leqslant m, \ \text{for all } x_1 \in [o, p]\bigr\}, \tag{5}$$

where the space $W_0^{2,\infty}(o, p)$ is given by

$$W_0^{2,\infty}(o, p) = \bigl\{f \in W^{2,\infty}(o, p): \ f(o) = f(p) = v \text{ and } f'(o) = f'(p) = 0\bigr\}.$$

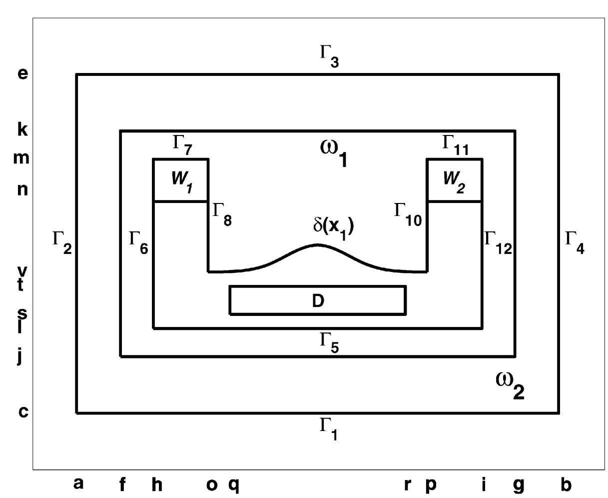

Fig. 1. Cross section of the electromagnet.