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Non-linear peristaltic transport of a Newtonian fluid in an inclined asymmetric channel through a porous medium

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Abstract

Peristaltic transport of an incompressible viscous fluid in an inclined asymmetric channel through a porous medium is studied under long-wavelength and low-Reynolds number assumptions. The flow is examined in a wave frame of reference moving with the velocity of the wave. The analytical solution has been obtained in the form of a stream function from which the axial velocity and pressure gradient have been derived. The results for the pressure drop and shear stress have also been computed numerically. The effects of various physical parameters are discussed through graphs and the phenomenon of trapping is also discussed. Comparison of various wave forms (namely sinusoidal, triangular, square and trapezoidal) on the flow is discussed.

Keywords: Peristalsis; Asymmetric channel; Froude number; Porous medium; Trapping

1. Introduction

Peristalsis is an important mechanism for mixing and transporting fluids, which is generated by a progressive wave of contraction or expansion moving on the wall of the tube. The mechanism of peristalsis occurs for urine transportation of chyme in the gastro-intestinal tract, the movements of the male spermatozoa in the ductus efferentes of the male reproductive tract, the movement of ovum in the fallopian tube and vasomotion in small blood vessels. The use of peristaltic pumping in biomedical devices like lung machine to pump the blood is also encountered. Peristaltic flows are exploited in industrial pumping as they provide an efficient means for sanitary fluid transport. The industrial use of peristaltic pumping in roller/finger pump is well known. Since the first investigation of Latham [1], a number of analytical, numerical and experimental [2–33] studies of peristaltic flows of different fluids have been reported under different conditions with reference to physiological and mechanical situations.

Flow through a porous medium has been studied by a number of workers employing Darcy law [13]. Varshney [14] studied the fluctuating flow of a viscous fluid through a porous medium bounded by porous and horizontal surface. Raptis et al. [15,16] studied the steady free convection and mass transfer flow of a viscous fluid through a porous medium bounded by a vertical surface. There are many examples of natural porous media, such as sand rye bread, wood, filter paper, bread, human lung, etc. A good biological situation of gallstones when they fall down into ducts and closed them partially or completely. Physiological, inflammation of the gallbladder epithelium often results from low-grade chronic infection, and this changes the absorptive characteristic of the gallbladder mucous. As a result, cholesterol begins to precipitate, usually forming many small crystals of cholesterol on the surface of the inflamed mucous membrane. These, in turn, act as nodes for further precipitation of cholesterol and crystals grow larger. When such crystals fall down the common bile ducts they cause loss of hepatic secretions to the gut, and also cause severe pain in the gallbladder region as well [29]. Recently, physiologists observed that the intra-uterine fluid flow due to myometrial contractions is peristaltic-type motion and the myometrial contractions may

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occur in both symmetric and asymmetric directions, De Vries et al. [31].

Eytan et al. [22] have observed that the characterization of non-pregnant woman uterine contractions is very complicated as they are composed of variable amplitudes, a range of frequencies and different wavelengths. It was observed that the width of the sagittal cross-section of the uterine cavity increases towards the fundus and the cavity is not fully occluded during the contractions. Recently, Eytan and Elad [23] have developed a mathematical model of wall-induced peristaltic fluid flow in a two-dimensional channel with wave trains having a phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross-section of the uterus. They have obtained a time dependent flow solution in a fixed frame by using lubrication approach. These results have been used to evaluate fluid flow pattern in a non-pregnant uterus. They have also calculated the possible particle trajectories to understand the transport of embryo before it gets implanted at the uterine wall. More recently, Srinivas and Pushparaj [32] have investigated the peristaltic pumping of MHD gravity flow of a viscous incompressible fluid in a two-dimensional asymmetric inclined channel having electrically insulated walls. The main purpose of the present study is to investigate the peristaltic pumping of incompressible Newtonian fluid in an inclined asymmetric channel through a porous medium. The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitudes and phase differences. The governing equations of fluid flow are solved subject to relevant boundary conditions. Peristaltic transport of intra-uterine fluid through a porous medium can be considered as an application of the present work. The comparison among four different wave forms, for the case of a symmetric porous channel, is also carefully made and the influence of several pertinent parameters on the stream function and pressure drop has been studied and numerical results obtained are presented.

2. Mathematical formulation and solution

We consider the motion of an incompressible viscous fluid in a two-dimensional inclined porous asymmetrical channel induced by sinusoidal wave trains propagating with constant speed c along the channel walls [Fig. 1]

$$Y = H_1 = d_1 + a_1 \cos\frac{2\pi}{\lambda}(X - ct) \qquad \text{upper wall},$$
$$Y = H_2 = -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \phi\right) \qquad \text{lower wall}, \quad (1)$$

where a_1, b_1 are the amplitudes of the waves, the channel is inclined at an angle α to the horizontal, λ is the wave length, $d_1 + d_2$ is the width of the channel, the phase difference ϕ varies in the range $0 \leqslant \phi \leqslant \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, and further a_1, b_1, d_1, d_2 and ϕ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \leqslant (d_1 + d_2)^2. \qquad (2)$$

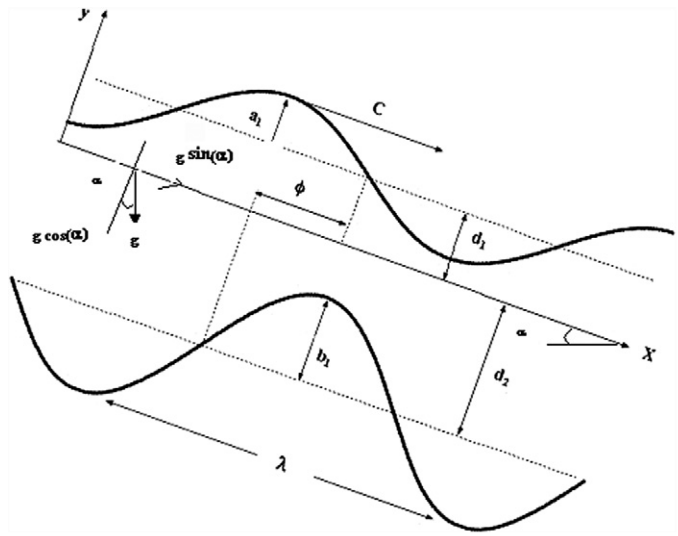

Fig. 1. Schematic diagram of the physical model.

3. Equations of motion

$$\rho\left[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right] = -\frac{\partial P}{\partial X} + \mu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\mu}{K}U + \rho g \sin\alpha, \qquad (3)$$

$$\rho\left[\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right] = -\frac{\partial P}{\partial Y} + \mu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\mu}{K}V - \rho g \cos\alpha, \qquad (4)$$

where U, V are the velocity components in the laboratory frame (X, Y), ρ is the density, μ is the coefficient of viscosity of the fluid, P is the pressure, g is the acceleration due to gravity, and K is the permeability parameter.

We shall carry out this investigation in a coordinate system moving with the wave speed, in which the boundary shape is stationary. The coordinates and velocities in the laboratory frame (X, Y) and the wave frame (x, y) are related by:

$$x = X - ct, \qquad y = Y, \qquad u = U - c,$$
$$\upsilon = V, \qquad p(x) = P(X, t), \qquad (5)$$

where u, υ are the velocity components in the wave frame (x, y), p and P are pressures in wave and fixed frame of references, respectively.

Introducing the following non-dimensional quantities:

$$\bar{x} = \frac{x}{\lambda}; \quad \bar{y} = \frac{y}{d_1}; \quad \bar{u} = \frac{U}{c}; \quad \bar{\upsilon} = \frac{V}{c\delta}; \quad \delta = \frac{d_1}{\lambda};$$
$$\bar{p} = \frac{d_1^2 P}{\mu c \lambda}; \quad \bar{t} = \frac{ct}{\lambda}; \quad h_1 = \frac{H_1}{d_1};$$
$$h_2 = \frac{H_2}{d_1}; \quad d = \frac{d_2}{d_1}; \quad a = \frac{a_1}{d_1}; \quad b = \frac{b_1}{d_1};$$
$$R = \frac{cd_1}{\nu}; \quad \bar{\psi} = \frac{\psi}{cd_1}; \quad \bar{K} = \frac{K}{d_1^2} \qquad (6)$$