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The multisoliton solutions for the nonisospectral mKPI equation with self-consistent sources

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Abstract

The nonisospectral mKPI equation with self-consistent sources is derived through the linear problem of the nonisospectral mKPI system. The bilinear form of the nonisospectral mKPI equation with self-consistent sources is given and the N-soliton solutions are obtained through Hirota method and Wronskian technique respectively.

Keywords: Nonisospectral mKPI equation with self-consistent sources; Hirota method; Wronskian technique

1. Introduction

Soliton equations with self-consistent sources [1–5] constitute an important class of integrable equations. They serve as important models fields of physics [4–6], such as hydrodynamics, solid-state physics, plasma physics. Besides, these kinds of systems also result in many mathematically interesting treatments and recently they were investigated by means of the inverse scattering transform, Darboux transformation, bilinear method, Wronskian technique, etc. [7–11]. Moreover, there exist many kinds of solutions to soliton equations with self-consistent sources, such as soliton, negaton, positon and complexiton solutions [12,13].

Nonisospectral soliton equations are also physical and mathematical importance. They are related to time-dependent spectral parameters [14,15]. Meanwhile, the time-dependent spectral parameters will lead to generalizations of those classical methods. In recent years, much attention has been paid on the study of nonisospectral soliton equations [16–18].

The Hirota method [19] and Wronskian technique [20] are two efficient approaches in finding exact solutions for soliton equations. Both of them are based on Hirota's bilinear form and consequently are called bilinear methods. Some soliton equations with self-consistent sources admit bilinear forms and N-soliton solutions in Hirota's expression. In addition, by means of a new determinant identity and a new verification procedure, their Wronskian solutions can also be derived. Recently, I study the mKPI equation with self-consistent sources [21] and nonisospectral mKPI equation [22] by Hirota method and Wronskian technique.

In this Letter, we would like to consider the nonisospectral mKPI equation with self-consistent sources (mKPIESCS) in a similar way to Ref. [17]. By use of compatible condition of linear problems, we lead out the nonisospectral mKPIESCS. Then we present a set of dependent variable transformations to write out the bilinear form of the nonisospectral mKPIESCS by which we can derive one- and two-soliton solutions successively through the standard Hirota's approach. On the basis of this, we conjecture further a general formula of N-soliton solution. Also we use Wronskian technique to give Wronski determinant of soliton solutions.

The Letter is organized as following. In Section 2, we lead out the nonisospectral mKPIESCS. In Section 3 the N-soliton solutions are given through Hirota's method. In Section 4 we show the solutions in Wronskian form.

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2. The nonisospectral mKPI equation with self-consistent sources

Consider the spectral problem and its adjoint associated with the nonisospectral mKPI equation

$$\Phi_y = B\Phi = \Phi_{xx} + 2u\Phi_x, \tag{1}$$

$$\Psi_y = -\Psi_{xx} + 2u\Psi_x. \tag{2}$$

Suppose that the time evolution of the eigenfunction Φ is given by

$$\Phi_t = C\Phi = \left[yA_3 + \frac{1}{2}xA_2 + \frac{1}{4}\partial + d\big(\Phi\Psi - \Phi\partial^{-1}\Psi_x\big)\right]\Phi, \tag{3}$$

$$A_3 = \partial^3 + 3u\partial^2 + \left(\frac{3}{2}\partial^{-1}u_y + \frac{3}{2}u_x + \frac{3}{2}u^2\right)\partial, \tag{4}$$

$$A_2 = \partial^2 + 2u\partial. \tag{5}$$

The compatibility of (1) and (3) requires that B, C satisfy

$$B_t = C_y - (C_{xx} + 2C_x\partial + 2uC_x + 2uC\partial - 2Cu\partial). \tag{6}$$

Substituting (1) and (3)–(5) into (6) and equating coefficients powers of ∂ and setting $d = 1$, we can obtain

$$4u_t + y\big(u_{xxx} - 6u^2u_x + 6u_x\partial^{-1}u_y + 3\partial^{-1}u_{yy}\big) + 2xu_y - u^2 + 3\partial^{-1}u_y + (\Phi\Psi)_x = 0. \tag{7}$$

This equation together with spectral problem (1) and (2) constitutes the nonisospectral mKPI equation with a self-consistent source. If taking $d = 0$, we can derive the nonisospectral mKPI equation [22]

$$4u_t + y\big(u_{xxx} - 6u^2u_x + 6u_x\partial^{-1}u_y + 3\partial^{-1}u_{yy}\big) + 2xu_y - u^2 + 3\partial^{-1}u_y = 0. \tag{8}$$

Generally, the nonisospectral mKP equation with N self-consistent sources can be defined as

$$4u_t + y\big(u_{xxx} - 6u^2u_x + 6u_x\partial^{-1}u_y + 3\partial^{-1}u_{yy}\big) + 2xu_y - u^2 + 3\partial^{-1}u_y + \sum_{j=1}^{N}(\Phi_j\Psi_j)_x = 0, \tag{9}$$

$$\Phi_{j,y} = \Phi_{j,xx} + 2u\Phi_{j,x}, \tag{10}$$

$$\Psi_{j,y} = -\Psi_{j,xx} + 2u\Psi_{j,x}, \tag{11}$$

where the operator C becomes

$$C = yA_3 + \frac{1}{2}xA_2 + \frac{1}{4}\partial + \sum_{j=1}^{N}\big(\Phi_j\Psi_j - \Phi_j\partial^{-1}\Psi_{j,x}\big). \tag{12}$$

3. Solving the nonisospectral mKPIESCS by Hirota method

In this section, we will solve the nonisospectral mKPIESCS (9)–(11) by the Hirota method. Through the transformations

$$u = \left(\ln\frac{g}{f}\right)_x, \qquad \Phi_j = \frac{h_j}{g}, \qquad \Psi_j = \frac{s_j}{f}, \tag{13}$$

the nonisospectral mKPESCS (9)–(11) can be transformed into the bilinear forms

$$D_x^2 g\cdot f - D_y g\cdot f = 0, \tag{14}$$

$$4D_t g\cdot f + y\big(D_x^3 g\cdot f + 3D_xD_y g\cdot f\big) + 2xD_y g\cdot f + g_x f + g f_x + \sum_{j=1}^{N} h_j s_j = 0, \tag{15}$$

$$D_y h_j\cdot f - D_x^2 h_j\cdot f = 0, \tag{16}$$

$$D_y s_j\cdot g + D_x^2 s_j\cdot g = 0, \tag{17}$$

where D is the well-known Hirota bilinear operator

$$D_x^l D_y^m D_t^n a\cdot b = (\partial_x - \partial_{x'})^l(\partial_y - \partial_{y'})^m(\partial_t - \partial_{t'})^n a(x,y,t)b(x',y',t')|_{x'=x,y'=y,t'=t}.$$