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Modified variational iteration method

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Abstract

In this Letter, we introduce a modified variational iteration method by inserting some unknown parameters into the correctional functional. The main advantage of this method is that one can avoid the uncontrollability problems, of the nonzero endpoint conditions, encountered in the traditional variational iteration method. Moreover, the method is applied to some nonlinear equations and the numerical solutions reveal that the modified method is accurate and efficient to solve a large class of nonlinear differential equations. Furthermore, the method does not share the drawbacks of the conventional variational iteration method, namely the restriction of the order of the nonlinearity term or even the form of the boundary conditions.

Keywords: Variational iteration method; Modified variational iteration method; Embedding parameters; Deformation; Elastic beam

1. Introduction

Many of engineering problems in the fields of heat transfer, vibration and oscillation lead to nonlinear equations which in the most cases are difficult to be solved analytically. Perturbation method, which is based on the existence of small/large parameters is being called perturbation quantity [1,2], is a well-known method to solve these problems. Many nonlinear problems do not contain such kind of perturbation quantity, and we can use nonperturbation methods, such as the artificial small parameter method [3], the δ-expansion method [4], the Adomian's decomposition method [5], the homotopy perturbation method (HPM) [6–10] and the variational iteration method (VIM) [11, 12]. In this Letter, we modify VIM and introduce a modified variational iteration method (MVIM) by inserting some unknown parameters into the correctional functional. The main advantage of this method is that one can avoid the uncontrollability problems of the nonzero endpoint conditions encountered in the traditional VIM. Moreover, the method is applied to some nonlinear equations and the numerical solutions reveal that MVIM is user friendly, flexible, accurate, effective and very powerful to solve a large class of differential equations. Furthermore, the method does not share the drawbacks of conventional VIM, namely the restriction of the order of the nonlinearity term or even the form of the boundary conditions.

2. Variational iteration method

2.1. Preliminaries

To illustrate the basic concepts of VIM, we consider the following differential equation [11]:

$$Lu + Nu = g(x) \tag{1}$$

where L, N and $g(x)$ are the linear operator, the nonlinear operator and a heterogeneous term, respectively. According to VIM, one can construct a correction functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\left\{Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\right\} d\tau \tag{2}$$

where the subscript n indicates the nth order approximation; $\tilde{u}_n$ is considered as a restricted variation, i.e. $\delta\tilde{u}_n = 0$ and λ is a general Lagrangian multiplier [12], which can be identified optimally via the variational theory. It is noteworthy that the

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variational of a functional is a first order approximation to the change in the functional at a particular value of the independent variable as we vary from curve to curve [13].

After imposing the variational to Eq. (2) we will have:

$$\delta\big(u_{n+1}(x)\big) = \delta\big(u_n(x)\big) + \delta\left(\int_0^x \lambda\big\{Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\big\}\,d\tau\right) \tag{3}$$

Taking restricted variation into consideration changes Eq. (3) into:

$$\delta\big(u_{n+1}(x)\big) = \delta\big(u_n(x)\big) + \delta\left(\int_0^x \lambda\big\{Lu_n(\tau)\big\}\,d\tau\right). \tag{4}$$

More effort makes the previous equation be more tangible, namely:

$$\delta\big(u_{n+1}(x)\big) = \delta\big(u_n(x)\big) + \lambda(\tau)\delta\left(\int_0^\tau \big\{Lu_n(\tau)\big\}\,d\tau\right)\Bigg|_{\tau=x} - \delta\left(\int_0^x \lambda'\delta\left(\int_0^\tau Lu_n(\tau)\,d\tau\right)d\tau\right). \tag{5}$$

In the foregoing, the order of the linear operator is considered as unity; otherwise the manipulation may take more time depending on the order of the operator.

Finally, if the unknown which is the linear operator L for any special problem is correctly defined, the corresponding Lagrangian multiplier will be identified.

It is worth noting that for linear cases ($N \equiv 0$) the first iteration of Eq. (2) leads to the exact solution; because; the exact Lagrangian multiplier is used while in nonlinear cases the most optimum one is considered.

2.2. Implementations

To show the efficiency and accuracy of the method, in the following we solve some examples by MVIM and compare the results with those of VIM and numerical methods.

2.2.1. Examples

Here we consider the second order differentiation equation with the second order nonlinearity term in the form of:

$$u_{xx} + u_x - \varepsilon u^2(x) = 0, \tag{6}$$

with the boundary conditions of:

$$u'(0) = 0, \qquad u(1) = 1 \tag{7}$$

where ε is a small parameter; while the subscript stands for the differentiation.

Now, to use VIM one has to make the correction functional relation for Eqs. (6) and (7), namely:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\big\{u_{XX} + u_X - \varepsilon \tilde{u}^2\big\}\,dX \tag{8}$$

where $\tilde{u}^2$ indicates the restricted variation; i.e. $\delta(\tilde{u}^2) = 0$. Making the above correction functional stationary, we have the following stationary conditions:

$$\lambda|_{X=x} = 0, \tag{9a}$$

$$\lambda + 1 - \lambda'|_{X=x} = 0, \tag{9b}$$

$$\lambda'' - \lambda' = 0. \tag{9c}$$

The Lagrangian multiplier can therefore be identified as:

$$\lambda = -1 + e^{X-x}. \tag{10}$$

Substituting the value of λ from Eq. (10) into Eq. (8) yields to the following iteration formula for VIM:

$$u_{n+1}(x) = u_n(x) + \int_0^x \big(-1 + e^{X-x}\big) \times \big\{u_{XX} + u_X - \varepsilon u^2\big\}\,dX. \tag{11}$$

To avoid the drawbacks of conventional VIM, namely; the uncontrollability of the nonzero endpoint conditions, we modify the method by introducing an unknown parameter (a), has to be multiplied to Eq. (11). Therefore, the iteration formula for MVIM for Eqs. (6) and (7) can take the form of:

$$u_{n+1}(x) = a\left(u_n(x) + \int_0^x \big(-1 + e^{X-x}\big) \times \big\{u_{XX} + u_X - \varepsilon u^2\big\}\,dX\right). \tag{12}$$

It is noteworthy to mention that the unknown a can take different values for different number of iterations. Therefore, it has to be determined for any iteration.

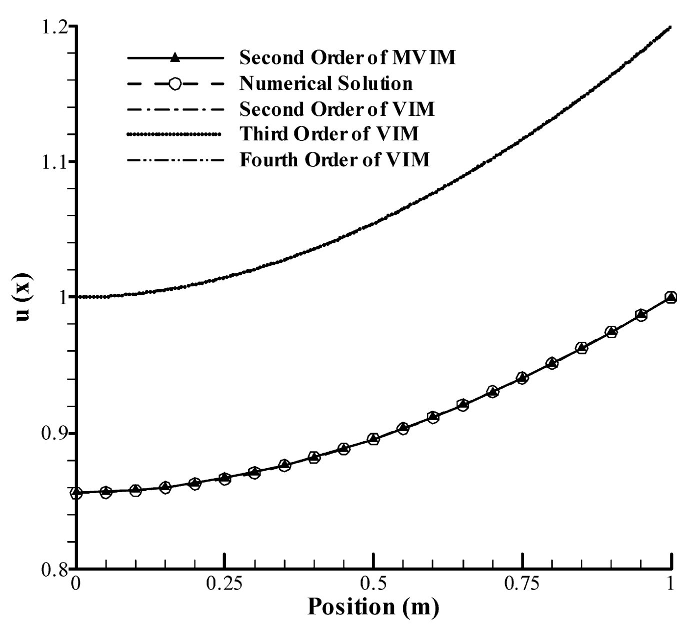

Fig. 1. The comparison of the results of the MVIM, VIM and the numerical solution assuming $\varepsilon = 0.5$ for Eqs. (6), (7).