Available online at www.sciencedirect.com
ScienceDirect
Physics Letters A 368 (2007) 181–187
PHYSICS LETTERS A
www.elsevier.com/locate/pla

General periodic flows of fractional Oldroyd-B fluid for an edge

S. Nadeem

Department of Mathematics, Quaid-i-Azam University, 45320 Islamabad, Pakistan

Received 18 May 2006; received in revised form 28 March 2007; accepted 2 April 2007

Available online 13 April 2007

Communicated by A.P. Fordy

Abstract

The velocity field corresponding to an incompressible fluid of fractional Oldroyd-B fluid subject to general periodic oscillations within an infinite edge are determined for all values of relaxation and retardation times. Based on the flow condition, the problem is solved by Fourier transform and Fourier sine transform and an exact solutions are obtained.

Keywords: Oldroyd-B fluid; Fractional derivatives; General periodic oscillations; Fourier transform; Fourier sine transform; Exact solutions

Introduction

In the literature, the mechanics of non-linear fluids presents special challenges to engineers, physicists and mathematicians since the non-linearity can manifest itself in a variety of ways. One of the simplest ways in which the viscoelastic fluids have been classified is the methodology given by Rivlin and Ericksen [1] and Truesdell and Noll [2], who present constitutive relations for the stress tensor as a function of the symmetric part of the velocity gradient and its higher (total) derivatives. In recent years there have been several studies [3–12] on flows of non-Newtonian fluids, not only because of their technological significance but also in the interesting mathematical features presented by the equations governing the flow. Since the non-Newtonian fluids are generally recognized more appropriate in industrial applications, the extension of plate and edge problem made an immediate headway in non-Newtonian fluids and a number of papers appeared in this direction [13–15]. Fetecau and Zierep [13] presented exact solution of a plate and an edge in second grade fluid which was then extended to a heated plate by Fetecau and Fetecau [16]. The solution for Maxwellian type of fluid was given by [14]. Fetecau [17] has also discussed the Rayleigh–Stokes problem for an edge in an Oldroyd-B fluid.

Recently, fractional calculus has encountered much success in the description of viscoelasticity [18–20]. The starting point of the fractional derivative model of non-Newtonian model is usually a classical differential equation which is modified by replacing the time derivative of an integer order by the so-called Riemann–Liouville fractional calculus operators. This generalization allows one to define precisely non-integer order integrals or derivatives. A very good agreement is achieved with experimental data when the Maxwell model is used with its first order derivatives replaced by the fractional-order derivatives [21]. With this in mind Wenchang et al. [22] discussed four flows of a visco-elastic fluid with fractional Maxwell model between two infinite parallel plates. Hayat et al. [23] discussed some unidirectional flows with fractional Maxwell and generalized second grade fluid.

The present analysis aims at finding the exact solution of an edge problem with fractional derivatives Oldroyd-B fluid. Such work seems to be important and useful because attention has been hardly given to the study of fractional Oldroyd-B fluid. To the authors knowledge such work does not seems to have undertaken in the case of an edge. The Oldroyd-B fluid which includes elastic and memory effects exhibited by dilute solutions, has been extensively used in many applications and also results of simulations fit experimental data quite well [24,25]. Keeping this all in mind exact solutions of fractional Oldroyd-B fluid due to an edge with

E-mail address: snqau@hotmail.com.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2007.04.032

general periodic oscillations has been established in this Letter. The flow due to certain special values of oscillations is then derived as a special case of periodic oscillations. The problem is solved by using Fourier transform of fractional derivatives and Fourier sine transform techniques.

Formulation of the problem

In this section we consider a fractional Oldroyd-B fluid which occupies the space on the rectangular edge defined by $-\infty < x' < \infty$, $y' \geqslant 0$, $z' \geqslant 0$. The flow is assumed to be generated by the periodic oscillations of the edge. The unsteady governing equation in two dimensions with appropriate boundary conditions is expressed as

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial^{\alpha+1} u}{\partial t^{\alpha+1}} = \nu \left(1 + \lambda_r \frac{\partial^\beta}{\partial t^\beta}\right)\left(\frac{\partial^2 u}{\partial y'^2} + \frac{\partial^2 u}{\partial z'^2}\right), \tag{1}$$

$$u(0, z', t) = u(y', 0, t) = U_0 f(t), \tag{2}$$

$$u(y', z', t) \to 0 \quad \text{as } {y'}^2 + {z'}^2 \to \infty, \tag{3}$$

where $f(t)$ are general periodic oscillations. The function $f(t)$ being periodic can be expressed as complex Fourier series $f(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t}$, where the Fourier coefficient a_k are given by

$$a_k = \frac{1}{T_0} \int f(t) e^{-ik\omega_0 t}\, dt \tag{4}$$

with non-zero fundamental frequency $\omega_0 = \frac{2\pi}{T_0}$. Here λ and λ_r are material constants referred to as relaxation and retardation time respectively, $\nu = (\mu/\rho)$ is the constant coefficient of viscosity and α and β are fractional calculus parameters. It should be noted that model includes the ordinary Oldrod-B model as a special case for $\alpha = \beta = 1$ and to second grade fluid when $\lambda = 0$, $\nu\lambda_r = \alpha_1/\rho$ and $\beta = 1$.

Introducing the nondimensional parameters

$$u = U_0 F, \qquad \tau = \frac{U_0^2 t}{\nu}, \qquad y = \frac{U_0 y'}{\nu}, \qquad z = \frac{U_0 z'}{\nu}, \qquad G = f.$$

We have the following nondimensional problem

$$\frac{\partial F}{\partial \tau} + \lambda_1 \frac{\partial^{\alpha+1} F}{\partial \tau^{\alpha+1}} = \left(1 + \lambda_2 \frac{\partial^\beta}{\partial \tau^\beta}\right)\left(\frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}\right), \tag{5}$$

$$F(0, z, \tau) = F(y, 0, \tau) = G(\tau), \tag{6}$$

$$F \to 0 \quad \text{as } y^2 + z^2 \to \infty,$$

where

$$\lambda_1 = \lambda \left(\frac{U_0^2}{\nu}\right)^\alpha, \qquad \lambda_2 = \lambda_r \left(\frac{U_0^2}{\nu}\right)^\beta,$$

$$G(\tau) = \sum_{k=-\infty}^{\infty} a_k e^{ink\tau}, \quad n = \frac{\omega_0 \nu}{U_0^2},\ a_k = \frac{1}{T_0} \int G(\tau) e^{ink\tau}\, d\tau.$$

Solution of the problem

In order to solve (5) subject to boundary conditions (6), we first apply the Fourier transform. So the temporal Fourier transform pair is defined as

$$M(y, z, \omega) = \int_{-\infty}^{\infty} F(y, z, \tau) e^{-i\omega\tau}\, d\tau, \tag{7}$$

$$F(y, z, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} M(y, z, \omega) e^{i\omega\tau}\, d\omega, \tag{8}$$

ω being the temporal frequency. We can obtain the Fourier transform of the fractional derivative [26].