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Nonlinear generation of sheared flows and zonal magnetic fields by electron whistlers in plasmas

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ABSTRACT

The nonlinear generation of shear field and flow in whistler waves is considered. It is shown that a coherent parametric process leads to modulational instability of four waves whistler interaction. Growth rates for the flow/field are compared with published simulation results.

1. Introduction

Theory of electron magnetohydrodynamic (EMHD) waves has been studied in a wide range of physical context in recent years. This is because such study is relevant in variety of physical problem starting from fast ignition concept of laser fusion in laboratory [1] to astrophysics [2], space physics, solar physics, etc. In spite of a long history of investigations there are many important issues which still attract attention to different interesting physical phenomena such as magnetic turbulence [3], fast magnetic field penetration in plasmas [4], reconnection of magnetic field [5] and many other problems [6]. EMHD normally describes the dynamics of electron fluid in presence of externally applied as well as self-generated magnetic field. The time scales in which such phenomena can occur are very short e.g. lying between inverse of electron and ion gyro-frequencies. In such a short time scale heavy ions remain unmagnetized in a static charge neutralizing background. Ions being immobile, the flow velocity of electrons determines current and hence is directly related to the curl of magnetic field. In the EMHD the careful inspection shows that the magnetic field evolves through the explicit nonlinear equation.

As we will see due to the presence of the nonlinearities, the EMHD dynamics becomes more complicated. A complete theory of the full nonlinear system is very difficult. However, a self-consistent numerical simulation is available in Ref. [7]. The question is: Can we do some analysis to get the deeper understanding of this numerical results? The answer to this question is: We do have some methods for treating certain aspects of complex nonlinear behavior of such EMHD plasma in a very simplest level. One of such methods is described in this study. The present day strategy for a better theoretical understanding is to apply different levels of perception and knowledge. An analytic numerical view will lead to a general feeling of which processes are important. More detailed and narrow views on specific processes are necessary for the understanding of the 'elementary processes'. They will thereby lead to an estimate of the potential the various 'elementary processes' do have in the overall dynamical evolution. In this work, we concentrate on 'simple models' which have been proven to be good candidates for modeling of some 'elementary processes'.

The generation and influence of velocity shear on whistler mode have been well investigated recently by Biskamp and his collaborators [7]. The results indicate that fully developed EMHD turbulence is characterized by double layer vorticity sheets and isolated circular monopolar vorticity eddies both of which constantly annihilated and reformed. Recently it has been shown that the nonlinear interaction between three whistler waves grows parametrically [8]. It may be the case that these EMHD vortices are themselves unstable to formation of shear. The observed process of shear field/flow generation, is the combination of several processes but the most 'robust' processes are flattening of magnetic field (flux) profile via nonlinear convection and generation of shear component via Reynolds stresses [9,10]. It appears that these processes seem to be irrelevant to the particular details of the magnetic field or flux function structures. In this work we report the mechanisms of self-generated flow and field to gain

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a better understanding in the theories of EMHD turbulence. As it is already mentioned such a model could help in scanning wider range of plasma parameters and elucidate the physics of the phenomenon [11,12].

The rest of the Letter is organized as follows. In Section 2, a simple physical model for EMHD waves is presented and the basic equations are derived. In Section 3 four wave coupling precesses are outlined and explicit growth rate for the modulational instability are obtained. Also in Section 3, numerical results are displayed in figures compared with simulations. We conclude in Section 4 with a discussion of our results.

2. Model and basic equations

The nonlinear fluid equations used to describe the electron mode are the electron momentum equations, current expressed in terms of electron velocity $\mathbf{J} = ne\mathbf{v}_e$ since ions are stationary at fast time scale. Finally magnetic field is closed by Maxwell's equation (Ampere's law) where we have ignored displacement current assuming $\omega \ll \omega_{pe}^2/\omega_{ce}$. In this Letter we have taken the vastly used and best illustrated model existing in Refs. [13,14]. The system described below is two-dimensional (i.e., $\partial/\partial z = 0$, no variation along the magnetic field). The nonlinear equations are derived with the following assumptions:

1. The time scale of the phenomena is very fast so that only electrons can participate in the dynamics and ions being heavy, they form a static charge neutralizing background. The spatial scale lies below the ion inertial scale c/ω_{pi}.
2. The density is assumed to be constant which is consistent with the incompressibility of electron flow, i.e., $\nabla\cdot\mathbf{v} = \nabla\cdot\mathbf{J} = 0$.
3. We restrict ourselves within two-dimensional (2D) configurations, where the magnetic field depends on two spatial coordinates (x, y) and on time t.
4. We assume plasma is weakly collisional and the beta value is low.

We start from electron momentum equation

$$m_e n\left(\frac{\partial}{\partial t} + \mathbf{v}_e\cdot\nabla\right)\mathbf{v}_e - ne\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e\times\mathbf{B}\right) - \nabla p_e - m_e n\nu_{ei}\mathbf{v}_e - m_e n\nu_e\nabla^2\mathbf{v}_e. \tag{1}$$

Here we assume that collisions are still frequent enough to make the pressure isotropic, and the divergence of the stress tensor is replaced by a scalar viscous diffusion term. The effective viscosity term used is very qualitative and the collisional diffusion is the effective energy sink and we have ignored the viscous dissipation. Using $\nabla\times\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t$ we obtain a self-consistent nonlinear equation for the magnetic field evolutions:

$$\frac{\partial}{\partial t}\left(1 - \delta_e^2\nabla^2\right)\mathbf{B} - \nabla\times\left[\mathbf{v}_e\times\left(1 - \delta_e^2\nabla^2\right)\mathbf{B}\right] = \frac{\eta c^2}{4\pi}\nabla^2\mathbf{B}, \tag{2}$$

where

$$\mathbf{v}_e = -\frac{\mathbf{J}}{en} = -\frac{c}{4\pi en}\nabla\times\mathbf{B}, \tag{3}$$

and $\delta_e = c/\omega_{pe}$ is the electron skin depth.

It is convenient to use the two scalar functions $\psi(x, y, t)$, which is the z component of the vector potential and $b(x, y, t)$, which is the z component of the magnetic field, instead of $\mathbf{B}$. Then we write $\mathbf{B} = \hat{e}_z\times\nabla\psi + b\hat{e}_z$. The evolution equations are

$$\left(\frac{\partial}{\partial t} + \hat{e}_z\times\nabla_\perp b\cdot\nabla_\perp\right)\left(\psi - d_e^2\nabla_\perp^2\psi\right) - \frac{\partial b}{\partial y} = \nu\nabla_\perp^2\psi, \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \hat{e}_z\times\nabla_\perp b\cdot\nabla\right)\left(b - d_e^2\nabla_\perp^2 b\right) + \hat{e}_z\times\nabla\psi\cdot\nabla_\perp\nabla_\perp^2\psi + \frac{\partial}{\partial y}\nabla_\perp^2\psi = \nu\nabla_\perp^2 b. \tag{5}$$

Note that the above equations have been written in normalized variables. The length scale is normalized by a typical scale length L and the magnetic field by a typical field intensity B_0. Therefore $\psi \equiv \psi/(B_0 L)$. Here $d_e = \delta_e/L$ is a dimensional parameter and δ_e is electron skin depth $\delta_e = c/\omega_{pe}$. The time is normalized by $(\Omega_e d_e^2)^{-1}$ where $\Omega_e(= eB_0/m_e c)$ is electron cyclotron frequency. A uniform magnetic field $B_0(\equiv d\psi_0/dx)$ in the y direction is assumed in deriving above equations and $\nu = \nu_{ei}/\Omega_e$.

The linear dispersion equation of the system is obtained by linearizing Eqs. (4) and (5) and assuming that the perturbed quantities vary as $\exp(ik_x x + ik_y y - i\omega t)$, where k_x and k_y are positive integers. The dispersion equation is given by

$$\omega = \omega_c\frac{k_y k_\perp\delta_e^2}{1 + k_\perp^2\delta_e^2}, \tag{6}$$

where $k_\perp^2 = k_x^2 + k_y^2$. This is the dispersion relation for whistler wave written in unnormalized variables. For $k\delta_e \ll 1$ we obtained $\omega = \omega_c k_y k_\perp\delta_e^2$. On the other hand, for $k\delta_e \gg 1$ we obtained $\omega = \omega_c k_y/k_\perp$. The later limit corresponds to electrostatic whistler wave. In the next section we study the nonlinear excitation of shear field and flow by four wave interactions.

3. Four wave interactions

To investigate the linear parametric excitation of shear field and flow by a pump whistler wave, the basic mode needed for this analysis is pump wave and in this analysis we assumed this is to be a whistler wave:

$$\xi_p = \xi_0 e^{-i\omega_0 t + ik_y y}. \tag{7}$$

Here ξ represents the axial (b) and poloidal magnetic field perturbations associated with the pump whistler wave. To investigate the stability of this wave to the generation of shear/zonal flow (b_s) and zonal magnetic field (ψ_s), the perturbed quantities can be represented as

$$\xi^s = \xi_s e^{-i\omega t + ik_x x}. \tag{8}$$

These two modes namely, the pump whistler wave and the shear mode, can couple to two sideband whistler waves as given below:

$$\xi^\pm = \xi_+ e^{i(\omega+\omega_0) + ik_x x + ik_y y} + \xi_- e^{i(\omega-\omega_0) + ik_x x - ik_y y}. \tag{9}$$

The wave number and frequency matching conditions determine the spatial and temporal dependence assigned to the $\pm$ sidebands. Because of the dispersive character of the whistler wave two sidebands are nonresonant and are equally important. This four wave coupling is the simplest model representing the generation of shear flow and field magnetic field by high frequency whistler wave. Here ω_0 is the mode frequency satisfying the dispersion relation for whistler waves

$$\omega_0 = \pm\frac{k_y^2}{1 + k_y^2 d_e^2}, \tag{10}$$

and the pump wave relationship

$$\psi^0 = -\left(\frac{k_y}{\omega_0}\right)\frac{b^0}{1 + k_y^2 d_e^2}, \tag{11}$$

$$b^0 = -\left(\frac{k_y}{\omega_0}\right)\frac{k_y^2\psi^0}{1 + k_y^2 d_e^2}. \tag{12}$$