Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics Letters A 353 (2006) 469–474
PHYSICS LETTERS A
www.elsevier.com/locate/pla

Compressible two-pressure two-phase flow models ☆

H. Jin [a,*], J. Glimm [b,c,1], D.H. Sharp [d,2]

[a] Department of Mathematics and Information, Research Institute for Basic Sciences, Cheju National University, Jeju, 690-756, Republic of Korea
[b] Department of Applied Mathematics and Statistics, Stony Brook University, Stony Brook, NY 11794-3600, USA
[c] Center for Data Intensive Computing, Brookhaven National Laboratory, Upton, NY 11793-6000, USA
[d] Applied Physics Division and Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Received 8 November 2005; accepted 15 November 2005

Available online 17 January 2006

Communicated by A.R. Bishop

Abstract

A central problem for compressible two-pressure two-phase flow models is closure, or the proper definition of averages of nonlinear terms. We propose here new closures for the velocity and momentum equations and discuss their validation.

MSC: 76T99; 76F25

Keywords: Multiphase flow; Hyperbolic models; Turbulence

1. Introduction

1.1. Summary of results

Multiphase flow has been studied for many decades [6,17, 19]. We discuss here compressible two-pressure two-phase flow models of the type proposed by Stewart and Wendroff [18], Ransom and Hicks [14], Chen et al. [3,11], and Saltz et al. [15]. The models governing the evolution of the fluid mixing are obtained by applying an appropriate averaging procedure to the microphysical equations [6]. They have distinct phase pressures and lead to hyperbolic models, eliminating mathematical difficulties of complex characteristics associated with single pressure flow models. The derivation of averaged equations leads to undefined averages of nonlinear functions of the primitive variables. These quantities must be modeled to close the system of equations.

The main result, achieved in part here, is to identify a closure which satisfies all the conservation and boundary constraints for the continuity and momentum equations, and which is validated against experimental or numerical data. In a second paper, we will close the energy equation and satisfy all constraints for this equation as well. To the authors' knowledge, previous two pressure closures did not achieve this goal. Based on the exact expressions for the interfacial terms, integral identities are proposed to define closures. The model proposed here is a modification of [2,3,5,11,15]; it is based on an assumed absence of internal length scales within the mixing zone; it assumes a mixing zone homogeneity.

The applicability of the model includes acceleration driven mixing processes. Acceleration driven mixing (e.g., Rayleigh–Taylor, Richtmyer–Meshkov instabilities) is characterized by well defined coherent structures which occupy the outer portions of the mixing layer. These are the bubbles of light fluid and the spikes of heavy fluid each penetrating into the opposite fluid type. The central portion of the mixing layer is typically

☆ This work was supported in part by US Department of Energy DE-FG02-90ER25084 and DE-AC02-98CH10886, National Science Foundation DMS-0102480, Army Research Office grant W911NF0510413 and Los Alamos National Laboratory. Los Alamos National Laboratory preprint number LA-UR-05-7229.
* Corresponding author. Tel.: +82 64 754 3567; fax: +82 64 725 2573.
E-mail addresses: hjin@cheju.ac.kr (H. Jin), glimm@ams.sunysb.edu (J. Glimm), dhs@t13.lanl.gov (D.H. Sharp).
[1] Tel.: +1 631 632 8552, +1 631 632 8355; fax: +1 631 632 8490.
[2] Tel.: +1 505 667 5266; fax: +1 505 665 3003.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2005.11.087

broken up into smaller structures. The phenomenology usually associated with multiphase flow, i.e., added mass and drag, is here taken from an assumed motion of the mixing zone edges, which are themselves governed by buoyancy drag equations, see [10–12]. With this as phenomenological input, we derive, in an essentially unique and exact manner, equations which govern the statistical averages of the coherent structures dominating the mixing zone edges. Beyond the input of velocities or trajectories of the edges of the mixing zone, this the model has no adjustable parameters. Implicit in this closure are the buoyancy, drag and added mass effects normally treated phenomenologically [2,6,9,13]. Form drag as a phenomenological term in the interior of the mixing zone is here replaced by p^* and $\Delta p = p_1 - p_2$ contributions to the pair of momentum equations. Indeed, Drew [6] speculated that a model with $\Delta p \neq 0$ should not include drag explicitly. Our results can be viewed as confirmation of his speculations.

1.2. The primitive equations

Let the function X_k be the phase indicator for material k ($k = 1, 2$); i.e., $X_k(t, \mathbf{x})$ equals 1 if position $\mathbf{x}$ is in fluid k at time t, zero otherwise. We average the advection law

$$\frac{\partial X_k}{\partial t} + v_{\text{int}} \cdot \nabla X_k = 0, \tag{1.1}$$

for the indicator function X_k of the region occupied by the fluid k. It was shown [6] that X_k satisfies (1.1). Here v_{int} is the microphysical velocity evaluated at the interface (the velocity component normal to the boundary ∂X_k is continuous so that $v_{\text{int}}\dot{\nabla} X_k$ is well defined). We also average the microscopic continuity and momentum equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0, \tag{1.2}$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \rho \mathbf{v}\mathbf{v} = -\nabla p + \rho g, \tag{1.3}$$

and one form of the energy equations

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot \rho \mathbf{v} E = -\nabla \cdot p\mathbf{v} + \rho \mathbf{v} g, \tag{1.4a}$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot \rho \mathbf{v} e = -p \nabla \cdot \mathbf{v}, \tag{1.4b}$$

$$\frac{\partial \rho S}{\partial t} + \nabla \cdot \rho \mathbf{v} S = 0. \tag{1.4c}$$

Here the dependent variables $\mathbf{v}$, ρ, p, e, E, and S denote, respectively, the velocity, density, pressure, internal energy, total energy, and entropy, with $E = e + \mathbf{v}^2/2$. These variables satisfy the thermodynamic relation

$$T\,dS = de + p\,d\left(\frac{1}{\rho}\right) \tag{1.5}$$

for smooth flows, where T is the temperature.

1.3. The averaged equations

The two-phase flow model presented here is a stochastic description of chaotic interpenetration of two inviscid non-heat-conducting fluids. It applies to flow regimes characterized by large scale coherent mixing structures (bubbles of light fluid, etc.), on the order of the thickness of the mixing zone, and by short time scales, so that relaxation terms are omitted. An ensemble average is applied to the micro equations (1.2)–(1.4c) to derive macro equations.

The notation used is as follows. The ensemble average is denoted $\langle \cdot \rangle$. To simplify the analysis, we consider a problem for which the ensemble has planar symmetry, so that the averaged equations are functions of a single variable, z. The average $\langle X_k \rangle$ of the indicator function X_k is denoted β_k; $\beta_k(z, t)$ is then the expected fraction of the horizontal layer at height z that is occupied by fluid k at time t. The quantities ρ_k and p_k are, respectively, phase averages of the density ρ and pressure p:

$$\rho_k = \frac{\langle X_k \rho \rangle}{\langle X_k \rangle}, \qquad p_k = \frac{\langle X_k p \rangle}{\langle X_k \rangle}. \tag{1.6}$$

The quantities v_k, e_k, E_k and S_k are, respectively, phase mass-weighted averages of the fluid z-velocity v_z, specific internal energy e, total energy E and entropy S:

$$v_k = \frac{\langle X_k \rho v_z \rangle}{\langle X_k \rho \rangle}, \qquad e_k = \frac{\langle X_k \rho e \rangle}{\langle X_k \rho \rangle},$$
$$E_k = \frac{\langle X_k \rho E \rangle}{\langle X_k \rho \rangle}, \qquad S_k = \frac{\langle X_k \rho S \rangle}{\langle X_k \rho \rangle}. \tag{1.7}$$

Applying the ensemble average to Eqs. (1.1)–(1.4c), we obtain the one-dimensional two-pressure two-phase flow averaged equations. We follow [2,3,6,16] to obtain

$$\frac{\partial \beta_k}{\partial t} + \langle \mathbf{v} \cdot \nabla X_k \rangle = 0, \tag{1.8}$$

$$\frac{\partial (\beta_k \rho_k)}{\partial t} + \frac{\partial (\beta_k \rho_k v_k)}{\partial z} = 0, \tag{1.9}$$

$$\frac{\partial (\beta_k \rho_k v_k)}{\partial t} + \frac{\partial (\beta_k \rho_k v_k v_k)}{\partial z} + \frac{\partial (\beta_k p_k)}{\partial z} = \left\langle p \frac{\partial X_k}{\partial z} \right\rangle + \beta_k \rho_k g, \tag{1.10}$$

for the advection of the volume fraction and for conservation of mass and momentum. We also have one and only one of the energy equations

$$\frac{\partial (\beta_k \rho_k E_k)}{\partial t} + \frac{\partial (\beta_k \rho_k v_k E_k)}{\partial z} + \frac{\partial (\beta_k p_k v_k)}{\partial z} = \langle p \mathbf{v} \cdot \nabla X_k \rangle + \beta_k \rho_k v_k g, \tag{1.11a}$$

$$\frac{\partial (\beta_k \rho_k e_k)}{\partial t} + \frac{\partial (\beta_k \rho_k v_k e_k)}{\partial z} + p_k \frac{\partial (\beta_k v_k)}{\partial z} = \langle p \mathbf{v} \cdot \nabla X_k \rangle, \tag{1.11b}$$

$$\frac{\partial (\beta_k \rho_k S_k)}{\partial t} + \frac{\partial (\beta_k \rho_k v_k S_k)}{\partial z} = 0, \tag{1.11c}$$

for the volume fraction β_k, velocity v_k, density ρ_k, pressure p_k, entropy S_k, internal energy e_k and total energy E_k of phase k. Here $g = g(t) > 0$ is the gravity.

Three interfacial terms are defined by

$$\langle \mathbf{v} \cdot \nabla X_k \rangle = v^* \frac{\partial \beta_k}{\partial z}, \tag{1.12}$$

$$\left\langle p \frac{\partial X_k}{\partial z} \right\rangle = p^* \frac{\partial \beta_k}{\partial z}, \tag{1.13}$$