Physics Letters A 374 (2010) 2383–2387

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Inelastic neutron scattering from superconducting rings

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ARTICLE INFO

Article history:
Received 17 March 2010
Accepted 25 March 2010
Available online 27 March 2010
Communicated by V.M. Agranovich

Keywords:
Neutron scattering
Superconducting ring
Differential cross section

ABSTRACT

For the first time the differential cross section for the inelastic magnetic neutron scattering by superconducting rings is derived taking account of the interaction of the neutron magnetic moment with the magnetic field generated by the superconducting current. Calculations of the scattering cross section are carried out for cold neutrons and thin film rings from type-II superconductors with the magnetic fields not exceeding the first critical field.

1. Introduction

Neutron scattering methods are widely used to study superconductors. Their magnetic excitations [1–3], dynamic and static heterogeneities [4–6], phonon spectra [7–9] have been investigated by using these methods. Also, small-angle neutron scattering techniques have been developed to determine both dynamic and static properties of the vortex lattices in the type-II superconductors in external magnetic fields exceeding the first critical field of the superconductor [10,11]. All of these neutron studies based on the well-known neutron scattering mechanisms in solids [12], were developed and applied for single-connected superconductors.

In this Letter we predict a new channel of inelastic magnetic neutron scattering from multiply-connected superconductors and, in particular, from superconducting rings in which, as is well known, the steady currents can flow in absent of an external magnetic field.

First of all, this inelastic scattering process which is due to the interaction of the neutron magnetic moment with the magnetic field generated by the superconducting current, is explained in detail. Then the differential cross section of this scattering process is derived that is defined by the off-diagonal matrix elements of the current density operator taken over the superconducting condensate wave functions in the ring. In the rest part of the work, quantitative calculations of the scattering cross section are carried out for cold neutrons and thin film rings from type-II superconductors with the film thickness $d < \lambda$ (λ is the penetration depth) and the magnetic fields generated by the currents in the film, not exceeding the first critical field of the superconductor. The temperature of the ring is assumed to be small as compared with T_c of the superconductor.

2. Hamiltonian of the system

Properties of a superconducting ring are completely determined by the initial condensate wave function ψ_m, where m is the number of the superconducting flux quanta (fluxoids) trapped in the ring. So, this wave function is defined both the quantization of the magnetic induction flux $\Phi_m = m\Phi_0$ and discrete values of the total current $J_m = \Phi_m/L$, where $\Phi_0 = \frac{h}{2e}$ is the fluxoid and L is the ring self-inductance.

A neutron incident on a superconducting ring with the superconducting current, creates the variable vector potential into the ring. Therefore, the interaction operator between the neutron and the ring can be written as $\hat{V} = \frac{1}{c}\int d\mathbf{r}\hat{\mathbf{A}}\hat{\mathbf{j}}$, where $\hat{\mathbf{A}}$ is the vector potential operator and $\hat{\mathbf{j}}$ is the current density operator in the ring. This interaction can also be presented in the completely identical form of the interaction of the neutron magnetic moment $\boldsymbol{\mu}_n$ with the magnetic field $\mathbf{B}$ generated by the superconducting current, $\hat{V} = \boldsymbol{\mu}_n\mathbf{B}$, as was shown in [13] for an arbitrary current.

Due to this interaction the kinetic energy of the neutron can change only discretely depending on the final number m_1 of the magnetic flux quanta trapped in the ring. This inelastic scattering process must be accompanied by a transition of the superconducting condensate from the initial state ψ_m to the final condensate state ψ_{m_1}.

The Hamiltonian of the neutron-superconducting ring system is:

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2010.03.057

$$H = \sum_m E_m a_m^+ a_m + \sum_{\mathbf{p},S} \varepsilon_{\mathbf{p}} c_{\mathbf{p}S}^+ c_{\mathbf{p}S} + \sum_{\mathbf{p},S,\mathbf{p}_1,S_1,m,m_1} V_{\mathbf{pp}_1}^{mm_1}(S,S_1) c_{\mathbf{p}_1 S_1}^+ c_{\mathbf{p}S} a_{m_1}^+ a_m, \qquad (1)$$

where a_m^+ (a_m) is the creation (destruction) operator of the m-state of the superconducting condensate in the ring; $\varepsilon_{\mathbf{p}}$, $\mathbf{p}$ and S are, respectively, the initial energy, wave vector and spin of the neutron; $\mathbf{p}_1$, S_1 are the final wave vector and spin of the neutron; $V_{\mathbf{pp}_1}^{mm_1}(S,S_1)$ is the matrix element of the interaction operator which has the form:

$$\hat{V} = 2\gamma \mu_N \hat{\mathbf{S}} \hat{\mathbf{B}}(\mathbf{r}_n). \qquad (2)$$

Here $\gamma = -1.91$; μ_N is the nuclear magneton; $\hat{\mathbf{S}}$ is the neutron spin operator; $\hat{\mathbf{B}}(\mathbf{r}_n)$ is the operator of the magnetic induction at the radius-vector of the neutron $\mathbf{r}_n$:

$$\hat{\mathbf{B}}(\mathbf{r}_n) = \frac{\mu_0}{4\pi} \int d\mathbf{r} \frac{[\hat{\mathbf{j}}(\mathbf{r}), \mathbf{r}_n - \mathbf{r}]}{|\mathbf{r}_n - \mathbf{r}|^3}. \qquad (3)$$

Below we use the cylindrical coordinates (z, ρ, φ) with the z-axis perpendicular to the plane of the ring, and assume that the initial wave vector $\mathbf{p}$ of the neutron is directed along the z-axis. Then the scattering cross section is independent from the polar angle φ due to the symmetry of the ring. In the Born approximation, the double differential cross section for this inelastic scattering is:

$$\frac{\partial^2 \sigma_{SS_1}}{\partial \varepsilon_{p_{1z}} \partial \varepsilon_{p_{1\rho}}} = 2^{-2} \pi^{-1} \frac{\Omega_n^2 m_n^2}{\hbar^4 \varepsilon_p^{1/2}} \sum_{m_1} \varepsilon_{p_{1z}}^{-1/2} \left| V_{\mathbf{pp}_1}^{mm_1}(S,S_1) \right|^2 \times \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_1} + E_m - E_{m_1}). \qquad (4)$$

Here $\varepsilon_{\mathbf{p}_1} = \frac{p_1^2}{2m_n} = \varepsilon_{p_{1z}} + \varepsilon_{p_{1\rho}}$ is the final energy of the neutron, m_n is the neutron mass, Ω_n is the normalizing volume of the neutron wave functions.

Using functions of the plane waves, it is easy to calculate the matrix element of (2) over the initial state of the neutron $\psi_{\mathbf{p}}$ and its final state $\psi_{\mathbf{p}_1}$. As a result, we obtain:

$$\hat{V}_{\mathbf{pp}_1} = \int d\mathbf{r}_n \psi_{\mathbf{p}_1}^* \hat{V} \psi_{\mathbf{p}} = -2i \frac{\gamma \mu_0 \mu_N}{\Omega_n q^2} \hat{\mathbf{S}} \left[\mathbf{q}, \int d\mathbf{r} \hat{\mathbf{j}}(\mathbf{r}) e^{i\mathbf{qr}} \right], \qquad (5)$$

where $\mathbf{q} = \mathbf{p} - \mathbf{p}_1$ is the momentum transfer.

The matrix element of the current density operator is written as:

$$\mathbf{j}_{mm_1} = \psi_{m_1}^* \hat{\mathbf{j}} \psi_m = \frac{ie\hbar}{m_C} (\psi_m \nabla \psi_{m_1}^* - \psi_{m_1}^* \nabla \psi_m) - \frac{4e^2}{m_C} \psi_{m_1}^* \hat{\mathbf{A}} \psi_m, \qquad (6)$$

where m_C is the mass of the Cooper pair with its charge equal to $2e$, $\hat{\mathbf{A}}$ is the operator of the vector potential which can be defined as:

$$\hat{\mathbf{A}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d\mathbf{r}_1 \frac{\hat{\mathbf{j}}(\mathbf{r}_1)}{|\mathbf{r} - \mathbf{r}_1|}.$$

For the superconducting ring $\mathbf{j}_{mm_1} = j_{mm_1}(z,\rho)\mathbf{i}_\varphi$. Then the matrix elements of the operator (5) taken over the condensate wave functions of the initial ψ_m and final ψ_{m_1} states in the ring, are given by:

$$\hat{V}_{\mathbf{pp}_1}^{mm_1} = 2i \frac{\gamma \mu_0 \mu_N}{\Omega_n q^2} \int d\mathbf{r}\, j_{mm_1}(z,\rho) e^{i\mathbf{qr}} \hat{\mathbf{S}}[\mathbf{i}_\varphi, \mathbf{q}]. \qquad (7)$$

Now we consider the matrix elements of (7) over the spin variables of the neutron. For polarized neutrons we obtain:

$$\langle \alpha | \hat{\mathbf{S}}[\mathbf{i}_\varphi, \mathbf{q}] | \alpha \rangle = -\frac{1}{2} q_\rho \cos(\varphi - \varphi_q),$$
$$\langle \beta | \hat{\mathbf{S}}[\mathbf{i}_\varphi, \mathbf{q}] | \beta \rangle = \frac{1}{2} q_\rho \cos(\varphi - \varphi_q) \qquad (8)$$

for the scattering without the spin flip process, and

$$\langle \beta | \hat{\mathbf{S}}[\mathbf{i}_\varphi, \mathbf{q}] | \alpha \rangle = \frac{1}{2} q_z e^{+i\varphi},$$
$$\langle \alpha | \hat{\mathbf{S}}[\mathbf{i}_\varphi, \mathbf{q}] | \beta \rangle = \frac{1}{2} q_z e^{-i\varphi} \qquad (9)$$

with the spin flip. Here φ_q is the polar angle of the scattering vector $\mathbf{q}$, $|\alpha\rangle$ and $|\beta\rangle$ are the spin functions with S_z equal to $+1/2$ and $-1/2$, respectively.

Substituting (7) and (8)–(9) in (4), the double differential cross section is reduced to the form:

$$\frac{\partial^2 \sigma}{\partial \varepsilon_{p_{1z}} \partial \varepsilon_{p_{1\rho}}} = 2^{-4} \pi^{-1} \gamma^2 \frac{e^2 \mu_0^2}{\hbar^2 \varepsilon_p^{1/2}} \sum_{m_1} \frac{j_{mm_1}^2(\mathbf{q})}{q^4 \varepsilon_{p_{1z}}^{1/2}} F(q) \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_1} + E_m - E_{m_1}), \qquad (10)$$

where $F(q) = q_\rho^2$,

$$j_{mm_1}(\mathbf{q}) = \int d\mathbf{r}\, j_{mm_1}(z,\rho) \cos(\varphi - \varphi_q) e^{i\mathbf{qr}} \qquad (11)$$

for the scattering without the spin flip process, and $F(q) = q_z^2$,

$$j_{mm_1}(\mathbf{q}) = \int d\mathbf{r}\, j_{mm_1}(z,\rho) e^{i\mathbf{qr} \pm i\varphi} \qquad (12)$$

with the spin flip.

Formula (10) is a general expression for the inelastic scattering cross section of neutrons by superconducting rings. To analyze (10), the off-diagonal matrix elements of the operator of the superconducting current density in the ring are required.

The diagonal matrix elements of the current density operator (6) or, in other words, the current distributions in superconducting rings were previously studied [14,15]. Typically, these results were obtained by numerical methods. However, these approaches do not allow to analyze clearly the dependence of the scattering cross section on the parameters of the superconductor, ring, and neutron energy. Below we consider a thin-film ring from the type-II superconductor, for which analytical expressions for the off-diagonal matrix elements of the superconducting current density can easily be obtained.

3. Off-diagonal matrix elements

Consider a rectangular cross-section ring from the type-II superconductor. The thickness of the ring obeys $d < \lambda$ (where λ is the London magnetic penetration depth), its inner radius $a \gg \lambda$ and outer radius $b \gg a$. In the cylindrical coordinates (z, ρ, φ) the ring is in the region $-d/2 \leqslant z \leqslant d/2$. We assume that the magnetic field in the ring is weak as compared with H_{c1} of the superconductor.

To find the magnetic induction in the ring, we use the London equation: $\Delta \mathbf{B} = \lambda^{-2} \mathbf{B}$. Since the ring thickness $d < \lambda$, the z-dependence of the magnetic induction can be neglected, and because of the circular symmetry, the magnetic field does not depend on the polar angle φ. As a result, we have the equation:

$$t^2 \mathbf{B}_{tt}'' + t \mathbf{B}_t' - t^2 \mathbf{B} = 0, \qquad (13)$$

where $t = \rho/\lambda$. In general, the solution of Eq. (13) is expressed in terms of the modified Bessel functions $I_o(\rho/\lambda)$ and $K_o(\rho/\lambda)$. The