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Energy transformation in creating dark solitons and sound waves

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ABSTRACT

The releasing of a Bose–Einstein condensate from a double well potential into a harmonic trap can induce interference patterns. We choose a soliton-like splitting barrier for the interference experiment. The numerical simulations show that dark solitons and sound waves are created directly when the collective movement of the condensate is weak. In particular, we calculate the energy of each production, and confirm that the energy of the region caused by the splitting barriers transfers into dark solitons and sound waves. Furthermore, based on the setup, we firstly indicate how much energy is transferred into the system and is apportioned into dark solitons.

1. Introduction

Since the experimental realization of a dilute atomic Bose–Einstein condensate (BEC) [1], the nonlinear properties of the matter waves have attracted extensive attention. One of the great interests is the creation of dark solitons [2–11]. Towards that goal, various methods have been developed, such as the phase imprinting [2,3] and the density engineering [6–9]. It is found that dark soliton, accompanied by a phase jump, results from a balance between the defocusing dispersion and the focusing repulsive nonlinear interaction. Recently, Weller et al. [12] have investigated the generation, subsequent oscillation and interaction of a pair of dark solitons by merging two coherent BECs initially prepared in a double well potential. The formation of solitons has been regarded as a consequence of interference [13–16]. Based on a dispersive hydrodynamic perspective, Hoefer et al. [17] have shown that the interference pattern between two BECs of sufficiently large density can be interpreted as a modulated soliton train. These works may lead to some confusion about the generation of interference fringes and the creation of dark solitons. As is well known, interference usually leads to the cosine-squared fringes, which differ from dark solitons [13–16]. Now one may want to know what causes the interference to create dark solitons or to produce fringes. Furthermore, many studies have mentioned the soliton energy, which can be computed theoretically [18–22]. However, few investigations have discussed where the soliton energy comes from. This problem is often neglected as the main attention is focused on the methods for generating the soliton as well as the stability of the soliton. For this, one may need to find out how much energy is transferred into the system as the method for creating solitons is applied. In addition, an experiment often not only produces dark solitons but also causes other things such as sound waves and the intense collective oscillation [2–11]. Hence, not all the energy is transferred into dark solitons. What is important here is whether the method is efficient for producing solitons.

In this study, by choosing a soliton-like splitting barrier, we have the usual matter–wave interference setup degenerate into a dark soliton generation. Such a design enables us to connect interference fringes with dark solitons and keep the whole condensate relatively static. Hence, it provides the possibility for us to examine the production exactly, and illustrate the resulting transfer of energy into each production. The numerical simulations show that the experiment can produce dark solitons and sound waves. Meanwhile, we illuminate the experiment by examining the energy of each production. It shows that the energy of the region, caused by the splitting barriers, changes into soliton energy and sound wave energy. Furthermore, our study firstly shows how much energy is transferred into the system as solitons are created and how much energy is apportioned into dark solitons. Also, the difference between our experiment and the usual interference of BEC in a harmonic trapped potential will be shown.

2. Basic equations and initial conditions for the experiments

Under strong transverse confinement, the BEC in the mean-field limit can be described by the one-dimensional Gross–Pitaevskii (GP) equation:

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$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=\left(-\frac{\hbar^2\nabla^2}{2m}+V_{\text{ext}}(x)+g\left|\psi(x,t)\right|^2\right)\psi(x,t). \quad (1)$$

Here, $\psi(x,t)$ denotes the macroscopic order parameter of the system, $V_{\text{ext}}(x)$ the confining potential, m the atomic mass, and $g = 4\pi\hbar^2 a_0/m$ the scattering amplitude, where a_0 is the s-wave scattering length.

We consider that the interference setup is made up of a combination of a harmonic trap and a middle barrier:

$$V_{\text{ext}}(x)=V_{\text{harmonic}}(x)+V_{\text{barrier}}(x)=\frac{m\omega^2x^2}{2}+V_{\text{barrier}}(x). \quad (2)$$

We suppose some interference that does not obviously excite the deformation of the main profile of the whole BEC or cause collision between BECs. Thus, the competition between repulsive interactions and the external confining potential will make the whole BEC relatively static. Even the interference will not break the main profile of the whole BEC. Can we obtain the interference? Generally speaking, the width of the barrier should be decreased to some critical value [12,15]. Below the value, the collective movement caused by the confining potential is feeble. We assume the middle barrier is:

$$V_{\text{barrier}}(x)=C\left[1-\left|\tanh(ax)\right|^b\right], \quad (3)$$

where a, b and C are used to tune the shape of the barrier. The barrier would only produce a localized defect in the BEC. Especially, if $C=\mu$, $a=1$ and $b=2$, the defect has the same shape as that of a static dark soliton in the Thomas–Fermi (TF) limit.

In numerical simulations, one may apply the density profile approximated by the TF solution as initial condition. To reduce the underlying density fluctuation at the edges, we use a smoothed initial profile:

$$n(x)=\max\left\{1-V_{\text{barrier}}(x)/\mu,0\right\}\cdot n_0(x), \quad (4)$$

where μ is the chemical potential of the atom. $n_0(x)$ is the density distribution of the ground state condensate in the harmonic trap, and it is obtained by propagating the TF wave function in imaginary time.

For the sake of simplicity, we set $\hbar=1$, the chemical potential $\mu=1$, the atomic mass $m=1$, and the scattering amplitude $g=1$. Thus the spatial extent of the system is characterized by the healing length $\xi=\hbar/\sqrt{m\mu}$, and the time unit is ξ/c, where the Bogoliubov speed of sound $c=\sqrt{\mu/m}$. Meanwhile, we have $\omega=(\sqrt{2}/100)(c/\xi)$ for the harmonic potential. Certainly, the barrier must split the BEC, i.e., $C\geqslant\mu$.

We now estimate the parameters for a realistic experiment. For a ^{23}Na condensate with $m=38.18\times10^{-27}$ kg and $a=2.8$ nm [23], we assume the tight transverse confining frequency $\omega_\perp = 5000\times2\pi$ Hz and the one-dimensional peak condensate density $n_{1D}=10^8$ m^{-1}. Thus, the longitudinal confining frequency is $39.7\times2\pi$ Hz. Our space and time units correspond to 0.4 μm and 5.7×10^{-5} s respectively. The system has the number of atoms $N_0\approx52000$.

3. Numerical experiments and results

We apply numerical simulation to show the experiments. We assume that the initial phase between the two parts of the BECs can be obtained by the phase-imprinting method. Meanwhile, only the middle barrier is removed at $t=0$. The harmonic well is kept to trap the BEC. We apply the expression $|\psi(x,t)|^2-n_0(x)$ to monitoring the relative fluctuation of the BECs. Figs. 1(a)–(d) present the results with one, two, three and four interference fringes, respectively. The position of the black lines denotes the fringes. The value of the maximal density fluctuation is 0.05 approximately. This means our experiments do not cause much deformation on the main profile of the BECs. Furthermore, the 'fringes' come together and attempt to reform the configuration of the initial defect, after that a new cycle starts [see Figs. 1(b)–(d)]. This indicates the 'fringes' pass through each other and regain their initial structures as the solitons with small amplitude in [24]. Therefore, the oscillation frequency of the 'fringes' approximates $\omega/\sqrt{2}$, which is the characteristic value of a dark soliton oscillating in the harmonically trapped BEC [12,18,19]. All these results indicate that the interference setup creates dark solitons and not simply 'fringes'.

In Figs. 1(b)–(d), the fluctuation which moves more rapidly than dark solitons is sound waves [2,3]. Generally, some sound waves are caused when dark solitons are created. Furthermore, there is another source of sound waves. As the solitons move, they become asymmetrically deformed and try to adjust to the inhomogeneous background by radiating counterpropagating sound waves [22].

For clarity, we plot the snapshots of the four cases at $t=150\xi/c$ [see Figs. 2(a)–(d)]. For comparison, we also plot the density profile in the presence of the barrier to each case (see the red curves). So we can see the solitons easily, i.e. the structure with density minima. The fluctuation is just the sound waves. In real experiments using phase-imprinting [2,3] or density engineering [6], dark solitons are often observed in association with sound waves. Thus, the numerical experiment is in qualitative agreement with the real one. For the center 'fringes' in Figs. 2(a) and (c), we find that the energy (1.33μ) is just equal to the value of a dark soliton with velocity $v=0$ [20,21]. The detailed calculation will be illuminated in the following text. Furthermore, we have checked that there is phase difference between the two sides of the solitons.

Why do the experiments create dark solitons and sound waves? It is easy to attribute the results to the nonlinear interference or to the collision of the two parts condensate. Here, to further explore the experiment, we consider the energy transformation. In [3], Denschlag et al. have distinguished dark soliton from sound waves according to its speed. Specially, Proukakis et al. [18,19] have monitored the change of the soliton energy to show the soliton–sound interactions. Now, a fundamental question arises naturally: where do the soliton energy and the sound wave energy come from in the above experiments? In fact, the remarkable change in our experiments is that the defect region would be broken. Therefore, a scenario is that the energy of the initial defect region transfers to dark solitons and sound waves. The next text will prove our assumption.

Let us first concentrate on the energy of dark solitons and sound waves. In the experiments, the energy of the soliton is calculated by integrating the GP energy function

$$\varepsilon(\psi)=\frac{\hbar^2}{2m}|\nabla\psi|^2+V_{\text{ext}}(x)|\psi|^2+\frac{1}{2}g|\psi|^4, \quad (5)$$

across the soliton region ($X\pm5\xi$, where X is the instantaneous position of the local density minimum) and subtracting the corresponding contribution of the background fluid [18,19,22]. Similarly, the energy of sound waves can be calculated across the sound wave region. Figs. 3(a) and (b) plot the temporal evolution of the energy of dark solitons and that of sound waves, respectively. The fluctuation of the energy in Figs. 3(a) and (b) shows the sound–soliton interaction. The more the sound wave energy is, the larger the fluctuation is. Meanwhile, the soliton energy is much larger than the sound wave energy in our experiments. Especially for the case in Fig. 1(a), the sound wave energy approaches zero. This point also means that the setup is very efficient for creating solitons. Furthermore, a very interesting property is that the total energy of sound waves and dark solitons is conserved [see Fig. 3(c)]. If the configurations are the same (see the two cases of $b=50$), the phases do not change the total energy.