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Optical solitons in some deformed MB and NLS–MB equations

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Abstract

In this Letter, we present the one- and two-soliton solutions for the nonlinear optical equations like Maxwell–Bloch and nonlinear Schrödinger–Bloch equation with pumping, by using auto Bäcklund transformation technique. These are the systems wherein we consider the variable spectral parameter. We also analyze the in-phase and off-phase two-soliton interaction for these systems and the results are discussed in detail.

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1. Introduction

To make the soliton based communication systems highly competitive and economical when compared to the conventional systems, attenuation in a fiber must be compensated. In this context, erbium-doped resonant silica fibers play a key role in not only minimizing the attenuation but also in achieving the goal of all-optical transmission with the help of experimentally observed phenomena like stimulated Raman scattering (SRS) and self-induced transparency [1,2]. The soliton pulse propagation in an erbium-doped fiber amplifier utilizes the self-induced transparency (SIT) phenomena, first discovered by McCall and Hahn [3].

The SIT phenomena pertains to the lossless pulse propagation in a resonant two level media such that when the energy difference between the two levels of the media coincide with the optical wavelength, then coherent absorption takes place and the media becomes optically transparent to that particular wavelength. As the energy

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difference between erbium atoms is found to be almost equal the soliton pulse wavelength, the fibers are generally doped with the erbium atoms in order to induce SIT phenomena. These rare earth ions doped fibers can be modeled as two level systems by considering only those two energy levels that participate in light induced transitions. The dynamic response of a two level system is governed by the well-known Maxwell–Bloch (MB) equations. There are many applications of MB equations to different physical problems such as SIT, self-focusing or resonant pulses and superfluoroscence, etc. These equations can be extended to the case of fiber amplifiers. So the fiber systems doped with erbium atoms is described by the coupled system of the nonlinear Schrödinger equation (NLS) along with the MB equations. Together they are known as the NLS–MB equation. When an optical pulse propagates through a nonlinear waveguide, the pulse evolution is governed by NLS–MB equations. The integrability aspects of NLS–MB system with variable dispersion, the study of propagation of optical solitons in coupled NLS–MB, and random nonuniform-doped media have been reported earlier wherein the spectral parameter was kept constant [4–10]. This is because the conventional methods of studying solitons by linear eigenvalue problem contain nonvariable spectral parameter λ i.e. assuming λ as a constant [10–15]. However, in recent years many-soliton equations with variable spectral parameter were reported already [10,12,14]. The nonlinear partial differential equations derived by IST with variable spectral parameter are also called as deformations. Burtsev et al. suggested a method where in the spectral parameter be regarded as a function of time, space and an additional complex constant called the 'hidden' spectral parameter. They called this method as IST with variable spectral parameter or the method of nonisospectral deformations [12,13]. Recently, using the covariance with respect to the Darboux transformation, Shin has systematically derived some soliton equation of inhomogeneous type [15].

In this Letter, we consider MB and NLS–MB systems with pumping and derive the one- and two-soliton solutions for these equations, which find many applications in nonlinear optics [4–6]. In the first part of this Letter, we discuss the one- and two-soliton solutions of MB equation with pumping and in the second part, we discuss the one- and two-soliton solutions of NLS–MB system. We also discuss the interaction scenario of two soliton in both the systems.

2. Maxwell–Bloch system with pumping

Now, we consider the following Maxwell–Bloch system with pumping [14],

$$\begin{aligned} \frac{\partial q}{\partial z} &= p, \\ \frac{\partial \eta}{\partial t} &= -4c - \frac{1}{2}(p q^* + p^* q), \\ \frac{\partial p}{\partial t} &= \eta q. \end{aligned} \tag{1}$$

In Eq. (1), for simplicity, we consider sharp line limit. Here, q represents the optical filed. p and η are polarization and population inversion given by $\nu_1\nu_2^*$ and $|\nu_1|^2 - |\nu_2|^2$, respectively. Here, ν_1 and ν_2 are the wave functions of the two energy levels of the resonant atoms and c defines pumping of an optical beam.

The eigenvalue problem for Eq. (1) is of the form:

$$\psi_t = U\psi, \qquad \psi_z = V\psi, \qquad \psi = (\psi_1, \psi_2)^T. \tag{2}$$

Here,

$$U = u_0 + i\lambda u_1, \qquad V = \frac{R}{4i\lambda}, \tag{3}$$

where

$$u_0 = \frac{1}{2}\begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix}, \qquad u_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} \eta & p \\ p^* & -\eta \end{pmatrix}. \tag{4}$$