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PHYSICS LETTERS A

Physics Letters A 348 (2006) 326–334

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Piecewise continuous distribution function method in the theory of wave disturbances of inhomogeneous gas

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Received 26 April 2005; received in revised form 12 July 2005; accepted 22 August 2005

Available online 31 August 2005

Communicated by F. Porcelli

Abstract

The system of hydrodynamic-type equations for a stratified gas in gravity field is derived from BGK equation by method of piecewise continuous distribution function. The obtained system of the equations generalizes the Navier–Stokes one at arbitrary Knudsen numbers. The problem of a wave disturbance propagation in a rarefied gas is explored. The verification of the model is made for a limiting case of a homogeneous medium. The phase velocity and attenuation coefficient values are in an agreement with former fluid mechanics theories; the attenuation behavior reproduces experiment and kinetics-based results at more wide range of the Knudsen numbers.

PACS: 05.20.Dd; 47.45.-n; 51.10.+y; 43.35.Ae

Keywords: Kinetic theory; Fluid mechanics; Rarefied gas dynamics; Ultrasound

1. Introduction

There is a significant number of problems of gas dynamics at which it is necessary to use a theory beyond the limits of traditional hydrodynamics of Navier–Stokes. The classical fluid mechanics is valid under the condition for the Knudsen number $Kn = l/L \ll 1$, where l is a mean free path, and L is a characteristic scale of inhomogeneity of a problem under consideration. The first work, in which wave perturbations of a gas was investigated from the point of view of more general kinetic approach (Boltzmann

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2005.08.054

equation), perhaps, is the paper of Wang Chang and Uhlenbeck [2].

Numerous researches on a sound propagation in a homogeneous gas at arbitrary Knudsen numbers were made, see, e.g., the classic experimental papers [3,4] and, may be the most advanced theoretical, kinetic-based [15]. The investigations have shown, that at arbitrary Knudsen numbers the behavior of a wave differs considerably from one predicted on a basis of hydrodynamical equations of Navier–Stokes. These researches have revealed two essential features: first, propagating perturbations keep wave properties at larger values of *Kn*, than it could be assumed on the basis of a hydrodynamical description. Secondly, at $Kn \geqslant 1$ such concepts as a wave vector and frequency of a wave become ill-determined.

The case, when the Knudsen number *Kn* is nonuniform in space or in time is more difficult for investigation and hence need more simplifications in kinetic equations or their model analogues. A constructions of such approaches for analytical solutions based on kinetic equation of Bhatnagar–Gross–Krook (BGK) or of Gross–Jackson [5] in a case of exponentially stratified gas were considered at [18,19,21,22] in connection with general fluid mechanics development. A progress was launched by more deep understanding of perturbation theory (so-called nonsingular perturbations), see, e.g., [17]. Recently the interest to the problem of *Kn* regime wave propagation has grown again [23–27].

In this Letter we develop the method of a piecewise continuous distribution function launched by ideas of von Karman, mentioned in a pioneering paper of Lees [9] and applied for a gas in gravity field in [21,22]. We consider the example of one-dimensional wave perturbations theory for a gas stratified in gravity field so that the Knudsen number exponentially depends on the (vertical) coordinate and generalize the results of earlier [21,22] to take into account the complete set of nonlinearities. We start with the method review and the generalization at the Section 2, go down to the linearized equations at the Section 3, deriving the dispersion relation at the Section 4 and, then, we study a solution of linear boundary problem to extract the attenuation parameter of the sound. At the Section 5 we pick up all the theoretical curves against the experimental data of Meyer [4] and Greenspan [3] and discuss the results.

2. Piecewise continuous distribution function method

The kinetic equation with the model integral of collisions in BGK form looks like:

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{r}} - g\frac{\partial f}{\partial v_z} = \nu(f_l - f), \tag{1}$$

here f is the distribution function of a gas, t is time, $\vec{v}$ is velocity of a particle of a gas, $\vec{r}$ is coordinate,

$$f_l(\vec{r},\vec{v},t) = \frac{n}{\pi^{3/2}v_T^3}\exp\left(-\frac{(\vec{v}-\vec{U})^2}{v_T^2}\right)$$

is the local-equilibrium distribution function, $v_T = \sqrt{2kT/m}$ denotes the average thermal velocity of particles of gas, $\nu = \nu_0\exp(-z/H)$ is the effective frequency of collisions between particles of gas at height z, $H = kT/mg$ is a parameter of the gas stratification. It is supposed, that density of gas n, its average speed $\vec{U} = (u_x, u_y, u_z)$ and temperature T are functions of time and coordinates.

Following the idea of the method of piecewise continuous distribution functions let us search for the solution f of Eq. (2) as a combination of two locally equilibrium distribution functions, each of which gives the contribution in its own area of velocities space:

$$f(t,\vec{r},\vec{V}) = \begin{cases} f^+ = n^+\left(\frac{m}{2\pi kT^+}\right)^{3/2}\exp\left(-\frac{m(\vec{V}-\vec{U}^+)^2}{2kT^+}\right), \\ \quad v_z \geqslant 0, \\ f^- = n^-\left(\frac{m}{2\pi kT^-}\right)^{3/2}\exp\left(-\frac{m(\vec{V}-\vec{U}^-)^2}{2kT^-}\right), \\ \quad v_z < 0, \end{cases} \tag{2}$$

here $n^\pm = n^\pm(t,z), U^\pm = U^\pm(t,z), T^\pm = T^\pm(t,z)$ are functional parameters of these locally equilibrium distributions functions.

Thus, a set of the parameters determining a state of the perturbed gas is increased twice. The increase of the number of parameters of distribution function (2) results in that the distribution function generally differs from a local-equilibrium one and describes deviations from hydrodynamical regime. In the range of small Knudsen numbers $l \ll L$ we automatically have $n^+ = n^-, U^+ = U^-, T^+ = T^-$ and distribution function (2) tends to local-equilibrium one, reproducing exactly the hydrodynamics of Euler and at the small difference of the functional “up” and “down” parameters—the Navier–Stokes equations. In