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Differential effective medium theory for thermal conductivity in nanofluids

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Abstract

By taking into account both physical and geometrical anisotropy of the highly conducting nanoparticle inclusions, we present differential effective medium theory to estimate the effective thermal conductivity in nanofluids. It is found that the adjustment of the nanoparticles shape is really helpful to achieve appreciable enhancement of effective thermal conductivity. Moreover, numerical results are in good agreement with the experimental ones observed in nanofluids. To one's interest, our theoretical predictions successfully show a nonlinear dependence of effective thermal conductivity on the volume fractions.

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Keywords: Nanofluids; Effective medium

Nanofluids, in which highly conductive solid nanoparticles with dilute volume fractions are randomly embedded in a quiescent fluid, are likely to be the future heat transfer media as their thermal conductivities are significantly higher than those of parent liquids [1–5]. The enhancement of thermal conductivity achieved in nanofluids is anomalously greater than the one predicted by conventional theories [2]. Following those experiment studies, various mechanisms and models have been proposed for explaining the anomalous enhancement of effective thermal conductivity such as the effect of the solid/liquid interfacial layer [6–8], the Brownian motion [9–11], and so on [12]. In addition, in view of the fact that carbon nanotubes possess large aspect ratio, Nan et al. generalized Maxwell–Garnett approximation to derive a quite simple formula for the effective thermal conductivity of carbon nanotube-based composites. The magnitude of large thermal conductivity enhancement observed in experiments was well predicted within Nan's model [13,14]. However, the model cannot explain the interesting phenomenon

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that the effective conductivity of nanotube suspensions is nonlinear with nanotube volume fractions. In this Letter, by taking into account the geometric anisotropy and the physical anisotropy simultaneously, we would like to present differential effective medium theory to investigate the effective thermal conductivity of nanofluids. The geometric anisotropy results from the large aspect ratio of carbon nanotube, while the physical anisotropy originates from the interfacial thermal resistance [14]. For randomly isotropic spherical inclusions, our new formula degenerates to the well-known differential one proposed by Bruggeman [15], which was applied to study the conductivity of a fluid-filled sandstone. We shall show that numerical results are in good agreement with those of experimental reports. More interestingly, even for extremely low volume fractions, we predict the nonlinear relation between the effective thermal conductivity and the volume fraction.

In the course of understanding the thermal transport behavior of nanofluids, we shall generalize Bruggeman differential effective medium theory [15] to estimate the effective thermal conductivity of nanofluids. First, we consider the composites in which randomly oriented spheroidal nanoparticles with low volume fractions f are embedded in a quiescent fluid with the thermal conductivity K_m. The assumptions that the particles are spheroidal in shape and are randomly oriented were already adopted in previous works [6,8,13,14]. Incidentally, our theory can be easily generalized to the composites of ellipsoidal particles. Due to the interfacial thermal barrier, the thermal conductivity of spheroidal particles must be anisotropic, denoted by K_x and K_z along transverse and longitudinal axes, respectively. Since the embedded spheroids are randomly oriented, the whole nanofluids will be isotropic. According to the traditional Maxwell–Garnett theory [16,17], the effective thermal conductivity K_e of composites can be expressed as,

$$K_e = K_m\left(1 + \frac{f}{3}(2\beta_x + \beta_z)\right), \tag{1}$$

where

$$\beta_x = \frac{K_x - K_m}{K_m + L_x(K_x - K_m)} \quad \text{and} \quad \beta_z = \frac{K_z - K_m}{K_m + L_z(K_z - K_m)},$$

here L_z [$L_x \equiv (1 - L_z)/2$] is the depolarization factor of spheroidal particles (z denotes the rotational axis) [16], given by

$$L_z = \begin{cases} \frac{1}{2p^3}(-2p + e\ln\frac{e-p}{e+p}) & \text{if } e > 1, \\ \frac{1}{2q^3}(2q - e\pi + 2e\arctan\frac{e}{q}) & \text{if } e < 1, \end{cases}$$

where e is the eccentricity, $p = \sqrt{e^2 - 1}$, and $q = \sqrt{1 - e^2}$. For different geometrical configurations of identical roughing spheres, L_z can be interpreted as equivalent depolarization factors. For example, one has $L_x = L_z = 0.435$ for a single-strand chain, and $L_x = 0.0865$, $L_z = 0.827$ for fcc lattice [18].

Starting with a homogeneous host material, we calculate the change in K_e from $K_e = K_m$ at $f = 0$ to $K_e + \Delta K_e$ at Δf,

$$\Delta K_e = K_m \frac{2\beta_x + \beta_z}{3}\Delta f. \tag{2}$$

To carry out further iterations, we simply reply K_m by K_e of the new homogenized composites and Δf with $\Delta f/(1 - f)$ due to the overlap effect. We arrive at the final differential equation,

$$\frac{\mathrm{d}K_e}{\mathrm{d}f} = \frac{K_e}{1 - f}\left[2\frac{K_x - K_e}{K_e + L_x(K_x - K_e)} + \frac{K_z - K_e}{K_e + L_z(K_z - K_e)}\right]. \tag{3}$$

Integrating the above equation and imposing the initial condition that $K_e = K_m$ at $f = 0$, we obtain

$$1 - f = \left(\frac{K_m}{K_e}\right)^{3A}\left(\frac{K_m + B_1}{K_e + B_1}\right)^{3C_1}\left(\frac{K_m + B_2}{K_e + B_2}\right)^{3C_2}, \tag{4}$$