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Formation of dust-acoustic shock waves in a strongly coupled cryogenic dusty plasma

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ABSTRACT

The nonlinear propagation of ultra-low-frequency dust-acoustic (DA) waves in a strongly coupled cryogenic dusty plasma has been investigated, by using the Boltzmann distributed electrons and ions, as well as modified hydrodynamic equations for strongly coupled charged dust grains. The reductive perturbation technique is used to derive the Burger equation. It is shown that strong correlations among negatively charged dust particles acts like a dissipation, which is responsible for the formation of the DA shock waves. The latter are associated with the negative potential, i.e. with the compression of negatively charged cryogenic dust particle density. It is also found that the effective dust-temperature, which arises from electrostatic interactions among negatively charged dust particles, significantly affects the height of the DA shock structures. New laboratory experiments at cryogenic temperature should be conducted to verify our theoretical prediction.

About two decades ago, Rao, Yu and Shukla [1] discovered the dust-acoustic (DA) waves in which the inertia is provided by the dust particle mass and the restoring force comes from the pressures of inertialess electrons and ions. The theoretical prediction of Rao, Yu and Shukla [1] has been conclusively verified by a number of laboratory experiments [2–5,8]. The linear properties of the DA waves, which are now found to be very common in both space and laboratory devices, are now well understood from both theoretical and experimental points of view [1,2,4–13].

The nonlinear DA waves have received a great deal of attention for understanding the properties of the localized electrostatic perturbations in space and laboratory dusty plasmas [10,14–19]. The nonlinear DA waves have been investigated theoretically [1,20–28] as well as experimentally [29–36] during the last few years. All of these theoretical and experimental investigations have been carried out at room temperature.

Recently, there has been some interest in investigating collective dust-plasma interactions [37–41]. Specifically, a number of new laboratory experiments [39–41] have been carried out at very low (cryogenic) temperatures of gas (liquid nitrogen or liquid helium or both) for studying the basic properties of cryogenic dusty plasmas (dusty plasmas at cryogenic temperatures in the range from 4.2 to 77 K). New cryogenic dusty plasma experiments have revealed that the cooling of thermal motions of ions down to cryogenic temperatures leads to decrease of the ion-Debye radius and increase of the dust number density (or decrease of the dust particle charge). Since in a cryogenic dusty plasma the dust particle number density is significantly high, the DA waves will not be subjected to dust-neutral collisions, contrary to those in a non-cryogenic dusty plasma at room temperature.

In this Letter, motivated by these new laboratory experiments [39–41], we investigate the nonlinear propagation of ultra-low frequency DA waves in a cryogenic dusty plasma which is composed of strongly coupled negatively charged mobile dust grains and Boltzmann distributed inertialess electrons and ions.

Let us consider the nonlinear propagation of ultra-low frequency DA waves in an unmagnetized cryogenic dusty plasma. At equilibrium, we have $Z_d n_{d0} + n_{e0} = n_{i0}$, where n_{d0}, n_{e0}, and n_{i0} are the unperturbed dust, electron, and ion number densities, respectively, and Z_d is the number of electrons residing on the dust grain surface. The nonlinear dynamics of the low phase velocity (in comparison with electron and ion thermal speeds) DA waves propagating in our cryogenic dusty plasma is governed by [42–44]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{1}$$

$$D_\tau \left(m_d n_d D_t u_d - Z_d e n_d \frac{\partial \phi}{\partial x} + T_* \frac{\partial n_d}{\partial x} \right) = \eta_l \frac{\partial^2 u_d}{\partial x^2}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e n_{e0} \left[\exp\left(\frac{e\phi}{T_e}\right) - \alpha \exp\left(-\frac{e\phi}{T_i}\right) + \frac{Z_d}{n_{e0}} n_d \right], \tag{3}$$

where n_d is the dust number density, u_d is the dust fluid speed, ϕ is the electrostatic wave potential, t (x) is the time (space) variable,

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T_e (T_i) is the electron (ion) temperature in units of the Boltzmann constant, m_d is the dust grain mass, e is the magnitude of the electron charge, $\alpha = n_{i0}/n_{e0}$, $D_\tau = 1 + \tau_m \partial/\partial t$, $D_t = \partial/\partial t + u_d \partial/\partial x$, τ_m is the viscoelastic relaxation time, η_l is the longitudinal viscosity coefficient, and T_* is the effective dust-temperature, which arises from the electrostatic interactions among highly charged dust grains. The effective dust-temperature T_* is defined as [43,44]

$$T_* = \frac{N_{nn}}{3} \frac{Z_d^2 e^2}{a_d} (1+\kappa) e^{-\kappa}, \tag{4}$$

where N_{nn} is determined by the dust structure [43,44], i.e. by the number of nearest neighbors (viz. in crystalline state $N_{nn} = 8$ for bcc lattice, $N_{nn} = 12$ for fcc lattice, etc.), $\kappa = a_d/\lambda_D$, a_d is the intergrain distance ($a_d \simeq n_{d0}^{-1/3}$), and λ_D is the screening length of the cryogenic complex plasma. There are various approaches for calculating the transport coefficients η_l and τ_m. These have been widely discussed in the literature [42,45–47].

To derive a dynamical equation for the nonlinear propagation of the ultra-low-frequency DA waves in a cryogenic complex plasma described by (1)–(3), we employ the reductive perturbation technique [48]. We, therefore, introduce the stretched coordinates [48,49]

$$\xi = \epsilon(x - V_p t), \qquad \tau = \epsilon^2 t, \tag{5}$$

where ϵ is a smallness parameter measuring the weakness of the dispersion, and V_p is the phase speed of the DA waves, and expand the variables n_d, u_d, and ϕ about the unperturbed states in power series of ϵ, viz.

$$n_d = n_{d0} + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots, \tag{6}$$

$$u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \cdots, \tag{7}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots. \tag{8}$$

Now, using (5)–(8) in (1)–(3), one can easily develop different sets of equations in various powers of ϵ. To the lowest order in ϵ [i.e. taking the coefficients of ϵ^2 from both sides of (1) and (2), and ϵ from both sides of (3)], one can obtain a set of coupled equations for $n_d^{(1)}$, $u_d^{(1)}$, and $\phi^{(1)}$. The latter can be solved to obtain

$$n_d^{(1)} = n_{d0} \frac{u_d^{(1)}}{V_p} = -\frac{Z_d n_{d0} e \phi^{(1)}}{m_d V_p^2 - T_*}, \tag{9}$$

$$\frac{V_p}{C_d} = \sqrt{\frac{\alpha - 1}{\alpha + \beta} + \gamma_*}, \tag{10}$$

where $C_d = (Z_d T_i/m_d)^{1/2}$, $\beta = T_i/T_e$, and

$$\gamma_* = \frac{T_*}{Z_d T_i} \equiv \frac{N_{nn}}{3} \frac{Z_d e^2}{a_d T_i} (1+\kappa) e^{-\kappa}. \tag{11}$$

Eq. (10) represents the linear dispersion relation for the DA waves in which the dust mass provides the inertia, and the electron and ion thermal pressures provide the restoring force. It is clear from (11) that the factor γ_* arises from the effective dust-temperature. To justify the importance of the effective dust-temperature T_*, one can estimate that $\gamma_* \simeq 0.5$ for cryogenic complex plasma environments [39–41], where $n_{i0} \simeq 10^9\ \text{cm}^{-3}$, $n_{d0} \simeq 9.9 \times 10^5\ \text{cm}^{-3}$, $Z_d \simeq 10^3$, $T_i \simeq 300$ K, and $T_e = 10T_i$. These parameters correspond to $\alpha = 10$, $\beta = 0.1$, and $C_d = 0.65$ cm/s. It clearly indicates that in a cryogenic complex plasma, the effective dust-temperature modifies the DA phase speed (V_p) significantly.

To the next order in ϵ [i.e. taking the coefficients of ϵ^3 from both sides of (1) and (2), and ϵ^2 from both sides of (3)], one can obtain another set of coupled equations for $n_d^{(2)}$, $u_d^{(2)}$, and $\phi^{(2)}$, which along with (9) and (10), reduce to a nonlinear dynamical equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{12}$$

where the nonlinear coefficient A and the dissipation coefficient C are given by

$$A = -f \sqrt{\frac{Z_d e^2}{m_d T_i}}, \tag{13}$$

$$C = \frac{1}{2} \frac{\eta_l}{n_{d0} m_d}, \tag{14}$$

$$f = \frac{C_d}{V_p} \left[1 + \frac{\alpha}{2} \left(\frac{1+\beta}{\alpha+\beta} \right)^2 + \frac{3}{2} \gamma_* \left(\frac{\alpha+\beta}{\alpha-1} \right) \right]. \tag{15}$$

Eq. (12) is the well-known Burger equation describing the nonlinear propagation of the ultra-low-frequency DA waves in the cryogenic complex plasma under consideration. It is obvious from (12) and (14) that the dissipative term, i.e. the right-hand side of (12) is due to the strong correlation among negatively charged cryogenic dust particles, and the term containing γ_* is due to the effective dust temperature, which arises from the electrostatic interactions among negatively charged cryogenic dust grains. It is clear that the nonlinear coefficient A is always negative, since $\alpha > 0$ and $f > 0$ are always satisfied.

We are now interested in looking for the stationary shock wave solution of (12) by introducing $\zeta = \xi - U_0 \tau'$ and $\tau' = \tau$, where U_0 is the shock wave speed (in the reference frame). This leads us to write (12), under the steady state condition ($\partial/\partial\tau' = 0$), as

$$-U_0 \frac{\partial \phi^{(1)}}{\partial \zeta} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}. \tag{16}$$

It can be easily shown that (16) describes the shock waves whose speed U_0 (in the reference frame) is related to the extreme values $\phi^{(1)}(-\infty)$ and $\phi^{(1)}(\infty)$ by $\phi^{(1)}(\infty) - \phi^{(1)}(-\infty) = 2U_0/A$. Thus, under the condition that $\phi^{(1)}$ is bounded at $\zeta = \pm\infty$, the shock wave solution of (16) is [50,51]

$$\phi^{(1)} = \phi_0^{(1)} \left[1 - \tanh(\zeta/\Delta) \right], \tag{17}$$

where $\phi_0^{(1)} = U_0/A$ and $\Delta = 2C/U_0$ are, respectively, the height and the thickness of the shock waves. It is obvious that the formation of such DA shock structures is due to the strong correlation among negatively charged cryogenic dust particles, and that such shock structures are associated with the negative potential ($\phi^{(1)} < 0$), i.e. with the compression of negatively charged cryogenic dust [as obvious from (9)]. It is clear that the shock thickness ($\Delta = \eta_l/U_0\rho_{d0}$) is directly proportional to the longitudinal viscosity coefficient (η_l), and is inversely proportional to the equilibrium dust grain mass density ($\rho_{d0} = n_{d0} m_d$).

We have numerically analyzed f [given by (15)] to show how the shock height [$\phi_0^{(1)}$] varies with different cryogenic dusty plasma parameters (viz. α, β, and γ_*). The variation of the shock height with α and β is obvious from the variation of $1/f$ with α and β, as shown in Fig. 1. On the other hand, the variation of the shock height with β and γ_* is obvious from the variation of $1/f$ with β and γ_*, as shown in Fig. 2. Fig. 1 shows that the height of the negative shock profiles decreases with β, but increases with α. We have also found that within typical ranges of cryogenic dusty plasma parameters [39–41] $\alpha = 2$–20, $\beta = 1$–10 and $\gamma_* = 0.1$–0.6, the shock height varies from $\sim 0.1T_i/e$ to $\sim 0.5T_i/e$, i.e. from 0.025 V to 0.125 V (for $T_i \simeq 0.025$ eV).

To summarize, the nonlinear propagation of ultra-low-frequency DA waves in a cryogenic complex plasma has been theoretically