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Excitation of multiple wakefields by short laser pulses in quantum plasmas

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ABSTRACT

We present a theoretical investigation of the excitation of multiple electrostatic wakefields by the ponderomotive force of a short electromagnetic pulse propagating through a dense plasma. It is found that the inclusion of the quantum statistical pressure and quantum electron tunneling effects can qualitatively change the classical behavior of the wakefield. In addition to the well-known plasma oscillation wakefield, with a wavelength of the order of the electron skin depth ($\lambda_e = c/\omega_{pe}$, which in a dense plasma is of the order of several nanometers, where c is the speed of light in vacuum and ω_{pe} is the electron plasma frequency), wakefields in dense plasmas with a shorter wavelength (in comparison with λ_e) are also excited. The wakefields can trap electrons and accelerate them to extremely high energies over nanoscales.

Three decades ago, Tajima and Dawson [1] demonstrated that intense laser pulses can efficiently generate electron plasma waves (EPWs) in their wake as they travel through a low density plasma. Physically, the ponderomotive force [2] of intense laser pulses pushes electrons locally in plasmas, which in turn oscillate at the electron plasma frequency with respect to the neutralizing background of immobile positive ions. The displacement of the electrons within the plasma gives rise to large electric fields, which can be much larger than any fields possible in a non-ionized material. An electron beam can surf on the electric field of a plasma wave picking up energy from the EPWs just as a surfer picks up energy from a water wave in the ocean [3]. The idea of Tajima and Dawson has now been experimentally verified worldwide [4–9].

Recently, there has been a great deal of interest in investigating the properties of high-energy density plasmas that are created by high intensity laser pulses. To probe dense matters, such as those also in the interior of white dwarf stars and Jovian planets, powerful laser-produced x-ray sources have been developed. They produce monoenergetic line radiation capable of penetrating through dense and compressed materials at solid density and above [10]. The x-ray measurement techniques are indicative of a dense Fermi-degenerate plasma state in laboratory. In a dense Fermi plasma, the electron degeneracy leads to the consideration of the Fermi–Dirac electron distribution and electron quantum tunneling through the quantum Bohm potential [11–13]. Furthermore, there also appear the spin force and spin magnetized electron current due to electron-1/2 spin in dense magnetoplasmas [14,15]. The quantum statistical pressure, the quantum Bohm force and the quantum spin force drastically affect the electron dynamics, and therefore one encounters numerous novel collective interactions in dense quantum plasmas. Specifically, it should be stressed that the quantum Bohm force effect, arising from the finite width of the electron wave function, gives rise to the dispersion of EPWs at nanoscales, which has important consequences to localized EPW structures [13,16] and plasmonic turbulences [17,18].

In this Letter, we present a theoretical investigation of the multiple EPW (wakefield) excitation by the ponderomotive force of a short laser pulse, accounting for the quantum statistical pressure and quantum Bohm force effects in the EPW dynamics. For our purposes, we shall use the quantum Madlung fluid equations [12], which is composed of the electron continuity and electron momentum equations, together with the Poisson equation, and derive the EPW (wakefield) equation in the presence of the radiation pressure. Choosing a specific form for the laser envelope, we solve the EPW equation analytically and numerically. We find that, due to the quantum Bohm force, there appear multiple wakefields at different nanoscales in dense quantum plasmas. In addition to the well-known wakefield, with a wavelength of the order of the electron skin depth, a short wavelength wakefield is also excited, with a scale length approaching the Compton wavelength. It turns out that for the case of a laser pulse in the optical regime, the short scale wakefield is suppressed. However, for short laser pulse lengths and/or high density plasmas, the energy density of the short scale wakefield may be comparable to that of the long wavelength wakefield. The consequences of our results are discussed.

We consider the propagation of a high-frequency laser pulse, with the vector potential $\mathbf{A} = \tilde{\mathbf{A}} \exp(ikx - i\omega t) + \text{c.c.}$, in an unmag-

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netized dense plasma. Here c.c. stands for the complex conjugate. The ponderomotive force of the high-frequency laser pulse drives longitudinal EPWs (wakefields) with a frequency much smaller than ω, but fast enough for the dynamics to take place on the electron timescale. The ions form a neutralizing background in our dense plasma. The governing equations for the wakefields are then the electron continuity equation

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}) = 0, \tag{1}$$

the electron momentum equation

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{e}{m}\nabla\phi - \frac{e^2}{2m^2c^2}|\tilde{\mathbf{A}}|^2 - \frac{V_F^2}{n_0}\nabla n_1 + \frac{\hbar^2}{4m^2}\nabla\nabla^2 n_1, \tag{2}$$

and the Poisson equation

$$n_1 = \frac{\nabla^2 \Phi}{4\pi e}, \tag{3}$$

where n_1 is the electron density perturbation in the equilibrium value n_0, $\mathbf{v}$ is the electron fluid velocity perturbation, ϕ is the wakefield potential, e is the magnitude of the electron charge, m is the electron rest mass, $V_F = (2\pi\hbar/\sqrt{3}m)(3n_0/8\pi)^{1/3}$ is the Fermi speed, and $\hbar$ is the Planck constant divided by 2π. We have thus assumed that the Fermi electron pressure dominates over the electron thermal pressure, appropriate for a high density plasma of moderate or low electron temperature. Several comments are in order. The second term in the right-hand side of (2), which represents the light pressure or the laser ponderomotive force, comes from the averaging (over the laser period) the advection and nonlinear Lorentz force in the electron equation of motion [2]. The fourth term in the right-hand side of (2) is the quantum Bohm force involving quantum electron tunneling in a dense quantum plasma.

Combining Eqs. (1)–(3) we obtain the plasma wakefield equation in the presence of the light pressure in our dense plasma

$$\left[\frac{\partial^2}{\partial t^2} + \omega_p^2 - v_F^2\nabla^2 + \frac{\hbar^2}{4m^2}\nabla^4\right]\Phi = \frac{\omega_p^2}{2c^2m}|\tilde{\mathbf{A}}|^2, \tag{4}$$

where $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency. In dense laboratory plasmas and in compact astrophysical objects, the latter is in the x-ray regime.

Let us now consider the excitation of one-dimensional wakefield by the high-frequency laser pulse that is propagating with the group velocity. Thus, we look for stationary solutions of (4) in a comoving frame. Letting $\xi = x - v_g t$, where v_g is the group velocity, we obtain from Eq. (4)

$$\left[\frac{\hbar^2}{4m^2}\frac{\partial^4}{\partial\xi^4} + \left(v_g^2 - v_F^2\right)\frac{\partial^2}{\partial\xi^2} + \omega_p^2\right]\Phi = \frac{\omega_p^2}{2c^2m}|\tilde{\mathbf{A}}|^2. \tag{5}$$

A simple special case is found when $v_g^2 = v_F^2$, which results in exponentially damped wakefields. However, below we will consider the case where we can have multiple oscillatory wakefields, which occur for plasmas of moderate density (see the condition below). We start by investigating the solutions to the left-hand side of Eq. (5) in the absence of the driving laser field. Making the ansatz $\Phi \propto \exp(ik\xi)$, we obtain

$$k^4 - k_a^2 k^2 + k_b^4 = 0,$$

where $k_a^2 = 4m^2(v_g^2 - v_F^2)/\hbar^2$ and $k_b^4 = 4m^2\omega_p^2/\hbar^2$. The solutions of (6) are

$$k^2 = +\frac{k_a^2}{2} \pm \sqrt{\frac{k_a^4}{4} - k_b^4},$$

unless the plasma density is very high (or $v_g^2 \simeq v_F^2$), $k_a^4/4 \gg k_b^4$ and the two solutions separate into oscillations with very different scale lengths. Denoting the solutions with k_+ and k_-, we have for $k_a^4/4 \gg k_b^4$

$$k_+^2 \approx k_a^2 = 4m^2\left(v_g^2 - v_F^2\right)/\hbar^2,$$
$$k_-^2 \approx k_b^4/k_a^2 = \omega_p^2/\left(v_g^2 - v_F^2\right).$$

The analysis below will be valid for arbitrary values of k_+ and k_-, however, as long as both modes are oscillatory, which holds whenever

$$v_g^2 > v_F^2 \quad \text{and}$$
$$m^2\left(v_g^2 - v_F^2\right)^2 > \hbar^2\omega_p^2$$

apply. When the lower inequality is a strong one, the oscillations at k_+ is close to the Compton scale (at least when $|v_g^2 - v_F^2| \simeq c^2$), and the k_- oscillations is the standard plasma oscillation wakefield, although somewhat modified by the inclusion of the Fermi pressure. As a starting point, however, no separation in magnitude of the two scales will be assumed.

Firstly, we rewrite Eq. (5) as

$$\left[\frac{\partial^4}{\partial\xi^4} + k_a^2\frac{\partial^2}{\partial\xi^2} + k_b^4\right]\Phi = \eta|\tilde{\mathbf{A}}|^2, \tag{6}$$

where $\eta = 2m\omega_p^2/\hbar^2c^2$. Eq. (6) can be integrated to yield

$$\Phi = \frac{\eta}{(k_+^2 - k_-^2)}\int_{-\infty}^{\xi}\left[\frac{1}{k_+}\sin\left[k_+(\xi - \xi')\right] - \frac{1}{k_-}\sin\left[k_-(\xi - \xi')\right]\right]\eta|\tilde{\mathbf{A}}|^2(\xi')\,d\xi', \tag{7}$$

where we have assumed the boundary condition that Φ is zero before the arrival of the high-frequency laser pulse.

Next, we assume that the laser pulse profile is a Gaussian, viz. $|A|^2 = A_0^2\exp(-\xi'^2/L^2)$. The energy density of the high-frequency laser pulse is assumed to be high enough such that the high-frequency laser pulse is changing its shape on a longer timescale, as compared to the wakefield generation process. The simplest case is that of a very short pulse, i.e. when $L \ll k_-^{-1}, k_+^{-1}$, when the wakefield after the laser pulse passage can be written as

$$\Phi = \Phi_+\sin(k_+\xi + \varphi_+) + \Phi_-\sin(k_-\xi + \varphi_-),$$

where φ_+ and φ_- are constant phase angles and the amplitudes of the wakefields are proportional to the high-frequency laser pulse energy, and are given by

$$\Phi_+ = \frac{\eta}{(k_+^2 - k_-^2)k_+}\int_{-\infty}^{\infty}|\tilde{\mathbf{A}}|^2(\xi')\,d\xi',$$

$$\Phi_- = \frac{\eta}{(k_+^2 - k_-^2)k_-}\int_{-\infty}^{\infty}|\tilde{\mathbf{A}}|^2(\xi')\,d\xi'.$$

In general, the wakefield amplitudes after the laser pulse passage can be written as

$$\Phi_\pm = \frac{\eta}{(k_+^2 - k_-^2)k_\pm}\int_{-\infty}^{\infty}\cos(k_\pm\xi)|\tilde{\mathbf{A}}|^2\,d\xi',$$

such that the electric field amplitudes $E_\pm$ of the different wakefields in the case of a Gaussian profile is

$$E_\pm = \frac{\eta}{(k_+^2 - k_-^2)}W\exp\left(-k_\pm^2L^2/4\right), \tag{8}$$