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Improved stability criteria for neural networks with time-varying delay ☆

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ABSTRACT

The problem of the stability analysis of neural networks with time-varying delay is considered in this Letter. By constructing a new augmented Lyapunov functional which contains a triple-integral term, an improved delay-dependent stability criterion is derived in terms of LMI using the free-weighting matrices method. The rate-range of the delay is also considered in the derivation of the criterion. Numerical examples are presented to illustrate the effectiveness of the proposed method.

1. Introduction

Recently, stability of the delayed neural network has been extensively studied. The existing stability criteria can be classified into two categories, namely, delay-independent ones [1–5] and delay-dependent ones [6–17]. Since delay-independent ones are usually more conservative than delay-dependent ones especially when the delay is small, delay-dependent stability criteria for delayed neural networks have received much attention.

In [6], the descriptor system approach was applied to derive the delay-dependent exponential stability conditions for delayed neural networks. An improved stability criterion was proposed in [7] by constructing a new Lyapunov functional and using the S-procedure. Using the free-weighting matrices method, a new delay-dependent stability criterion for neural networks with time-varying delay is derived in [8]. In the above papers, some useful terms were ignored when estimating the upper bound of the derivative of the Lyapunov functional. So some less conservative stability criteria were proposed in [9] by considering those useful terms and using the free-weighting matrices method. The stability of neural networks with time-varying interval delay were considered in [18] where the relationship between the time-varying delay and its lower and upper bound was taken into account and an elegant result was derived. However, there still exists room for further improvements.

It can be seen that the Lyapunov functional introduced in [18] only contains some integral terms, for example $\int_{t-h}^{t} z^T(s)Qz(s)\,ds$, and double-integral terms, for example $\int_{-h}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)Z\dot{z}(s)\,ds\,d\theta$. If a triple-integral term is introduced in the Lyapunov functional, what results can be obtained? This idea motivates this study. In addition, the lower bound of the delay-derivative was not considered in the above publications. If information on the lower bound of the delay-derivative is used in the derivation of stability criterion, a less conservative result may be obtained.

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2. Problem formulation

Consider the following delayed neural network:

$$\dot{x}(t) = -Cx(t) + Ag\big(x(t)\big) + A_1 g\big(x\big(t-\tau(t)\big)\big) + u \tag{1}$$

where $x(\cdot) = [x_1(\cdot)\ x_2(\cdot)\ \cdots\ x_n(\cdot)]^T$ is the neuron state vector, $g(x(\cdot)) = [g_1(x_1(\cdot))\ g_2(x_2(\cdot))\ \cdots\ g_n(x_n(\cdot))]^T$ is the neuron activation function, and $u = [u_1\ u_2\ \cdots\ u_n]^T$ is a constant input vector. $C = \operatorname{diag}\{c_1, c_2, \ldots, c_n\}$ with $c_i > 0$, $i = 1, 2, \ldots, n$, is a diagonal matrix representing self-feedback term, A is the connection weight matrix and A_1 is the delayed connection weight matrix. The delay $\tau(t)$ is a time-varying differentiable function satisfying

$$0 \leqslant \tau(t) \leqslant h \tag{2}$$

and

$$\mu_1 \leqslant \dot{\tau}(t) \leqslant \mu_2 < 1 \tag{3}$$

where $h \geqslant 0$, μ_1 and μ_2 are constants. It is assumed that each neuron activation function, $g_i(\cdot)$, $i = 1, 2, \ldots, n$, is nondecreasing, bounded and satisfying the following condition:

$$0 \leqslant \frac{g_i(x) - g_i(y)}{x - y} \leqslant m_i \quad \forall x, y \in \mathbb{R},\ x \neq y,\ i = 1, 2, \ldots, n, \tag{4}$$

where m_i, $i = 1, 2, \ldots, n$, are positive constants.

Assuming that $x^* = [x_1^*\ x_2^*\ \cdots\ x_n^*]$ is the equilibrium point of (1) whose uniqueness has been given in [17] and using the transformation $z(\cdot) = x(\cdot) - x^*$, (1) can be converted to the following error system:

$$\dot{z}(t) = -Cz(t) + Af\big(z(t)\big) + A_1 f\big(z\big(t-\tau(t)\big)\big) \tag{5}$$

where $z(\cdot) = [z_1(\cdot)\ z_2(\cdot)\ \cdots\ z_n(\cdot)]^T$ is the state vector, $f(z(\cdot)) = [f_1(z_1(\cdot))\ f_2(z_2(\cdot))\ \cdots\ f_n(z_n(\cdot))]^T$, and $f_i(z_i(\cdot)) = g_i(z_i(\cdot) + x_i^*) - g_i(x_i^*)$, $i = 1, 2, \ldots, n$. According to (4), one can obtain that the functions $f_i(\cdot)$, $i = 1, 2, \ldots, n$, satisfy the following condition:

$$0 \leqslant \frac{f_i(z_i)}{z_i} \leqslant m_i, \quad f_i(0) = 0,\ \forall z_i \neq 0,\ i = 1, 2, \ldots, n, \tag{6}$$

which is equivalent to

$$f_i(z_i)\big[f_i(z_i) - m_i z_i\big] \leqslant 0, \quad f_i(0) = 0,\ i = 1, 2, \ldots, n. \tag{7}$$

3. Main results

In this section, a new augmented Lyapunov functional is constructed and a less conservative delay-dependent stability criterion is obtained.

Theorem 1. *For given scalars $h \geqslant 0$, μ_1 and μ_2, system (5) is asymptotically stable for any time-varying delay satisfying (2) and (3) if there exist matrices $R_l = R_l^T > 0$, $S_l = S_l^T > 0$, $l = 1, 2$, $U = U^T > 0$, $P = P^T = [P_{ij}]_{4\times4} > 0$, $Q = Q^T = [Q_{ij}]_{3\times3} > 0$, $X = X^T = [X_{ij}]_{7\times7} \geqslant 0$, $\Lambda = \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} \geqslant 0$, $W_k = \operatorname{diag}\{W_{1k}, W_{2k}, \ldots, W_{nk}\} \geqslant 0$, $k = 1, 2$, and any matrices L, N, and H with appropriate dimensions such that the following LMIs holds*:

$$\begin{bmatrix} \Xi & A_c^T Y & \frac{1}{2}h^2 H \\ Y A_c & -Y & 0 \\ \frac{1}{2}h^2 H^T & 0 & -\frac{1}{2}h^2 U \end{bmatrix} < 0, \tag{8}$$

$$\Pi_1 = \begin{bmatrix} X & \Gamma + H & L \\ \Gamma^T + H^T & S_1 & 0 \\ L^T & 0 & S_2 \end{bmatrix} \geqslant 0, \tag{9}$$

$$\Pi_2 = \begin{bmatrix} X & \Gamma + H & N \\ \Gamma^T + H^T & S_1 & 0 \\ N^T & 0 & S_2 \end{bmatrix} \geqslant 0 \tag{10}$$

where

$$\Xi = [\Xi_{ij}]_{7\times7},$$

$$\Xi_{11} = -P_{11}C - C^T P_{11} - Q_{12}C - C^T Q_{12}^T + P_{14} + P_{14}^T + Q_{11} + R_1 + hS_1 + L_1 + L_1^T + hH_1 + hH_1^T + hX_{11},$$

$$\Xi_{12} = -L_1 + L_2^T + N_1 - C^T P_{12} + P_{24}^T + hH_2^T + hX_{12},$$

$$\Xi_{13} = P_{12} + L_3^T + hH_3^T + hX_{13},$$

$$\Xi_{14} = -C^T P_{13} + P_{34}^T + L_4^T - P_{14} - N_1 + hH_4^T + hX_{14},$$

$$\Xi_{15} = P_{13} + L_5^T + hH_5^T + hX_{15},$$

$$\Xi_{16} = P_{11}A + Q_{12}A + Q_{13} - C^T Q_{23} + L_6^T + hH_6^T - C^T \Lambda + MW_1 + hX_{16},$$