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Attracting and invariant sets of fuzzy Cohen–Grossberg neural networks with time-varying delays ☆

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ABSTRACT

In this Letter, a class of fuzzy Cohen–Grossberg neural networks (FCGNNs) with time-varying delays is investigated. With removing some restrictions on the amplification functions, a new nonlinear delay differential inequality is established, which improves previously known criteria. By using the properties of M-cone and a generalization of Barbălat's lemma, the boundedness and asymptotic behavior for the solution of the inequality are obtained. Applying this nonlinear delay differential inequality, a series of new and useful criteria are obtained to ensure the existence of global attracting set and invariant set for FCGNNs with time-varying delays. An example is given to illustrate the effectiveness of our results.

1. Introduction

Since Cohen–Grossberg neural network (CGNN) was first proposed by Cohen and Grossberg [1] in 1983, many researchers have done extensive works on this subject due to their extensive applications in many fields such as pattern recognition, parallel computing, associative memory, signal and image processing and combinatorial optimization. In such applications, it is of prime importance to ensure that the designed neural networks is stable.

In reality, time delays often occur due to finite switching speeds of the amplifiers and communication time. Moreover, it was observed both experimentally and numerically that time delay could destroy a stable network and cause sustained oscillations, bifurcation or chaos, and thus could be harmful. In recent years, the dynamical behaviors of Cohen–Grossberg neural networks with delays have been studied by many researchers [2–6].

However, besides delay effects, in mathematical modeling of real world problems, we will encounter some other inconveniences, for example, the complexity and the uncertainty or vagueness. Fuzzy theory is considered as a more suitable setting for the sake of taking vagueness into consideration. Based on traditional cellular neural networks (CNNs), Yang and Yang proposed the fuzzy CNNs (FCNNs) [7,8], which integrates fuzzy logic into the structure of traditional CNNs and maintains local connectedness among cells. Unlike previous CNNs structures, FCNNs have fuzzy logic between its template input and/or output besides the sum of product operation. FCNNs are very useful paradigm for image processing problems, which is a cornerstone in image processing and pattern recognition. Recently, some results on stability and other behaviors have been derived for fuzzy neural networks with or without time delays, see [9–13]. Therefore, it is necessary to consider both the fuzzy logic and delay effect on dynamical behaviors of neural networks. Nevertheless, to the best of our knowledge, the stability analysis for FCGNNs with time-varying delays has seldom been investigated and remains important and challenging. It is well known that Cohen–Grossberg neural network model is one of the most popular and typical neural network models.

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Some models, such as Hopfield-type neural networks, CNNs and bidirectional associative memory neural networks, are special cases of Cohen–Grossberg neural network model. So it is not only theoretically interesting but also practically important to determine asymptotic stability for FCGNNs with time-varying delays.

Motivated by the above discussions, the main purpose of this Letter is to study the global attracting set and invariant set of FCGNNs with time-varying delays. Different from [14–16], in this Letter, we will introduce a new nonlinear differential inequality, which is more effective than the linear differential inequalities for studying the asymptotic behavior of some nonlinear differential equations. Applying this new nonlinear delay differential inequality, the attracting set and invariant set of FCGNNs are obtained. Meanwhile, using the properties of M-cone and a generalization of Barbălat's lemma, the boundedness and asymptotic behavior for the solution of the inequality are obtained. Furthermore, without using Lyapunov functional, the proposed method is shown to be simple yet effective for analyzing the asymptotic behavior of FCGNNs with time-varying delays.

2. Preliminaries

Let E denote the n-dimensional unit matrix, $\mathcal{N} \triangleq \{1, 2, \dots, n\}$, and $\mathbb{R}_+ \triangleq (0, \infty)$. For $A, B \in \mathbb{R}^{m\times n}$ or $A, B \in \mathbb{R}^n$, $A \geqslant B$ $(A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\geqslant$ $(>)$".

$C[X, Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. In particular, let $C \triangleq C[[-\tau, 0], \mathbb{R}^n]$ denote the family of all bounded continuous $\mathbb{R}^n$-valued functions ϕ defined on $[-\tau, 0]$ with the norm $\|\phi\| = \sup_{-\tau\leqslant\theta\leqslant 0} |\phi(\theta)|$, where $|\cdot|$ is Euclidean norm of $\mathbb{R}^n$.

For any $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $\varphi \in C$, we define

$$[x]^+ = \big(|x_1|, \dots, |x_n|\big)^T \triangleq \mathrm{col}\big\{|x_i|\big\}, \qquad [A]^+ = \big(|a_{ij}|\big)_{n\times n},$$
$$\big[\varphi(t)\big]_\tau = \mathrm{col}\big\{\big[\varphi_i(t)\big]_\tau\big\}, \qquad \big[\varphi(t)\big]_\tau^+ = \big[\big[\varphi(t)\big]^+\big]_\tau, \qquad \big[\varphi_i(t)\big]_\tau = \sup_{-\tau\leqslant\theta\leqslant 0} \big\{\varphi_i(t+\theta)\big\}.$$

For an M-matrix D [17, p. 114], we denote $D \in \mathscr{M}$ and $\Omega_M(D) \triangleq \{z \in \mathbb{R}^n \mid Dz > 0,\ z > 0\}$.

In this Letter, we will study:

$$\begin{cases} \dot{x}_i(t) = \alpha_i(x_i(t))[-\beta_i(x_i(t)) + \sum_{j=1}^n \gamma_{ij}\mu_j + I_i + \bigwedge_{j=1}^n a_{ij}g_j(x_j(t)) + \bigwedge_{j=1}^n b_{ij}g_j(x_j(t-\tau_{ij}(t))) + \bigwedge_{j=1}^n T_{ij}\mu_j \\ \qquad + \bigvee_{j=1}^n c_{ij}g_j(x_j(t)) + \bigvee_{j=1}^n d_{ij}g_j(x_j(t-\tau_{ij}(t))) + \bigvee_{j=1}^n H_{ij}\mu_j], \quad t \geqslant t_0, \\ x_i(t_0+s) = \phi_i(s), \quad -\tau \leqslant s \leqslant 0,\ i \in \mathcal{N}, \end{cases} \tag{1}$$

where $x_i(t)$ is the ith neuron state, α_i represents an amplification function, β_i is an appropriately behaved function, g_j denotes the activation function, $\tau_{ij}(t)$ with $0 \leqslant \tau_{ij}(t) \leqslant \tau$ (τ is a constant, $i, j \in \mathcal{N}$) are the transmission delays, γ_{ij} are elements of fuzzy feed-forward template, a_{ij} and b_{ij} are elements of fuzzy feedback MIN template, c_{ij} and d_{ij} are elements of fuzzy feedback MAX template, T_{ij} and H_{ij} are elements of fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively. μ_i and I_i denote input and bias of the ith neuron, respectively.

To prove our results, the following lemmas are necessary.

Lemma 2.1. *(See [18, Yang and Yang].) For any $a_{ij} \in \mathbb{R}$, $x_j, y_j \in \mathbb{R}$, $i, j \in \mathcal{N}$, we have the following estimations*:

$$\left|\bigwedge_{j=1}^n a_{ij}x_j - \bigwedge_{j=1}^n a_{ij}y_j\right| \leqslant \sum_{j=1}^n |a_{ij}||x_j - y_j|, \qquad \left|\bigvee_{j=1}^n a_{ij}x_j - \bigvee_{j=1}^n a_{ij}y_j\right| \leqslant \sum_{j=1}^n |a_{ij}||x_j - y_j|.$$

We also need the following generalization of Barbălat's lemma.

Lemma 2.2. *(See [19, Aizerman and Gantmakher].) Let $f(t)$ be defined, continuous, and piecewise continuously differentiable for $t \geqslant 0$, and let $f(t)$ and $\dot{f}(t)$ be bounded*; *let $G(x)$ be defined, continuous, and positive-definite for all x. Further, let*

$$\int_0^\infty G\big(f(t)\big)\,dt < \infty,$$

then $f(t) \to 0$ as $t \to \infty$.

Throughout the Letter, we always assume that for any $\phi \in C$, system (1) has least one solution through (t_0, ϕ), denoted by $x(t, t_0, \phi)$ or $x_t(t_0, \phi)$ (simply $x(t)$ and x_t if no confusion should occur), where $x_t(t_0, \phi) = x(t+s, t_0, \phi) \in C$, $s \in [-\tau, 0]$.

Definition 2.1. A set $S \subset C$ is called a positive invariant set of system (1), if for any initial value $\phi \in S$, we have $x_t(t_0, \phi) \in S$, $t \geqslant t_0$.

Definition 2.2. A set $S \subset C$ is called a global attracting set of system (1), if for any initial value $\phi \in C$, the solution $x_t(t_0, \phi)$ converges to S as $t \to \infty$. That is,

$$\mathrm{dist}(x_t, S) \to 0 \quad \text{as } t \to \infty,$$

where $\mathrm{dist}(\varphi, S) = \inf_{\psi\in S} \mathrm{dist}(\varphi, \psi)$, $\mathrm{dist}(\varphi, \psi) = \sup_{s\in[-\tau,0]} |\varphi(s) - \psi(s)|$, for $\varphi \in C$.