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Soliton and chaotic structures of dust ion-acoustic waves in quantum dusty plasmas

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ABSTRACT

The quantum hydrodynamic model is employed to study the soliton and chaotic structures of dust ion-acoustic waves in quantum dusty plasmas consisting of electrons, ions, and negatively/positively charged dust particles. By means of the reductive perturbation technique, two-dimensional Davey–Stewartson (DS) system is derived. By improving the extended projective method and the extended tanh-function method, a separation of variables solution with arbitrary functions for the Davey–Stewartson system is obtained. Many soliton and chaotic structures such as localized nonlinear coherent structure, line-soliton structure, periodic wave pattern structure, Rössler and Lorenz chaotic structures are given. It is found that these structures are effected by the quantum effects.

1. Introduction

Quantum plasmas have gained much interests in ultrasmall electronic devices [1], dense astrophysical environments [2], ultracold plasmas [3] and laser plasmas [4]. In quantum plasmas, the de-Broglie wavelength of the charge carriers, i.e., $\lambda_B = h/(2\pi m v)$, becomes comparable to the spatial scale of the system, so the quantum effects are expected to play a crucial role in plasma dynamics. Many authors have studied linear and nonlinear wave propagation in quantum plasmas. For instance, Refs. [5–8] have studied the dispersion properties and nonlinear dynamics of unmagnetized and magnetized quantum plasmas. Shukla and Stenflo [9] have investigated the dispersion properties of the shear Alfvén modes in homogeneous ultracold quantum magnetoplasmas. Among the nonlinear structures in quantum plasmas, both dust acoustic waves (DAW) and dust ion-acoustic waves (DIAW) have been investigated in Refs. [10,11]. Most of these work have studied the quantum DAW and quantum DIAW by using the perturbation technique and various nonlinear equations are obtained such as Korteweg–de Vries (KdV) [10], Kadomtsev–Petviashvili (KP) equation [12], Zakharov–Kuznetsov (ZK) equation [13], nonlinear Schrödinger equation (NLSE) [14] and so on. In fact, considering a different scale transform, one can also obtain Davey–Stewartson (DS) system, which governs the dynamics of the nonlinear modulated wave packets [15].

Therefore, in this Letter, we employ the QHD model to study the small-amplitude, two-dimensional DIAW in a quantum dusty plamas (QDP) and obtain the DS system. By using a new method which combines the advantages of both extended projective method [16–19] and extended tanh-function [20], we successfully obtain a separation of variables solution for the DS system. Many types of soliton structures such as the localized nonlinear coherent structure, line-soliton structure, periodic wave pattern structure, etc., are obtained. On the other hand, chaotic dynamics has been intensively investigated with the help of simple low-dimensional models such as Lorenz [21] system and the Rössler system [22] in plasmas physics [23–26]. For instance, a novel hyperchaos is obtained in the quantum Zakharov system

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for plasmas [24]. Chaotic behavior of relativistic electron motion in a free-electron laser with realizable helical wiggler and axial magnetic field is investigated by using Poincaré maps and Liapunov exponents [26]. Therefore, in this Letter, we try to study the chaotic structures with the help of different chaotic system for the DIAW in QDP.

This Letter is organized in the following fashion. In Section 2, we present the QHD model to study the DIAW in unmagnetized QDP. In Section 3, by means of the reductive perturbation technique, we study the modulated DIAW and obtain DS system governing the evolution of the wave envelope. In Section 4, we solve the DS system by a new method and obtain many types of soliton structures and chaotic structures for the modulated DIAW in quantum dusty plasmas. The summary is given in Section 5.

2. Basic equations

Let us study the two-dimensional obliquely propagating dust ion-acoustic waves (DIAW) in a three-species quantum dusty plasmas (QDP), whose constituents are dust particles, inertial ions with a background stationary dust of constant charged while the electrons are taken inertialess. DIAW in such a quantum plasma system is described by the following normalized equations:

$$\begin{cases} \frac{\partial n_i}{\partial t}+\frac{\partial(n_i u)}{\partial x}+\frac{\partial(n_i v)}{\partial y}=0,\\ \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial\phi}{\partial x}=0,\\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial\phi}{\partial y}=0,\\ \nabla^2\phi=\beta n_e-\alpha N_{d0}-n_i, \end{cases} \tag{1}$$

where $\nabla=\hat{x}\partial/\partial x+\hat{y}\partial/\partial y$. n_i and u, v are the number density and the fluid velocity of the ion particles normalized by n_{i0} and the quantum ion-acoustic velocity $C_{si}=\sqrt{2K_B T_{\mathrm{Fe}}/m_i}$, respectively. n_e is the number density of the electrons normalized by n_{e0}. ϕ is the electrostatic wave potential normalized by $2K_B T_{\mathrm{Fe}}/e$. The space and time coordinates x, y and t are normalized by the quantum Debye length $\lambda_D=\sqrt{2K_B T_{\mathrm{Fe}}/4\pi n_{i0}e^2}$ and the ion plasma period $\omega_{pi}^{-1}=\sqrt{m_i/4\pi n_{i0}e^2}$, respectively. Further, $\alpha=\pm1$ for positive/negative dust particles, $\beta=n_{e0}/n_{i0}$ so that $\beta=1+\alpha N_{d0}$ with $N_{d0}=z_{d0}n_{d0}/n_{i0}$. The primes from normalized quantities have been dropped and the terms proportional to m_e/m_i have been disregarded in Eqs. (1) in the limit $m_e/m_i\ll1$.

We assuming that the electrons in a 2D Fermi plasma follow the pressure law $p_e=(m_e V_{\mathrm{Fe}}^2/2n_{e0})n_e^2$, where $V_{\mathrm{Fe}}=\sqrt{2K_B T_{\mathrm{Fe}}/m_e}$ is the Fermi speed [27]. Therefore, the inertialess electron momentum equation reads

$$\nabla\phi-\nabla n_e+\frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)=0, \tag{2}$$

where $H_e=\sqrt{\hbar^2\omega_{pi}^2/m_e m_i C_{si}^4}$ is the non-dimensional quantum diffraction parameter.

Integrating Eq. (2) once with boundary conditions $n_e=1$ and $\phi=0$ at $\pm\infty$, we obtain

$$\phi-n_e+1+\frac{H_e^2}{2\sqrt{n_e}}\nabla^2\sqrt{n_e}=0. \tag{3}$$

3. Derivation of Davey–Stewartson system

In order to study the modulated DIAW in QDP with transverse perturbations, we apply the reductive perturbation technique (RPT) to Eqs. (1) and (3). Different from the Sagdeev potential approach, the RPT is a well-known method mostly applied to small-amplitude nonlinear waves. We introduce the independent variable as $\xi=\epsilon(x-ct)$, $\eta=\epsilon y$, $\tau=\epsilon^2 t$.

The dependent variables are expanded as [15]

$$\begin{cases} n_i=1+\sum_{n=1}^{\infty}\epsilon^n\sum_{l=-\infty}^{\infty}n_{il}^{(n)}(\xi,\eta,\tau)e^{i(kx-wt)l},\\ u=\sum_{n=1}^{\infty}\epsilon^n\sum_{l=-\infty}^{\infty}u_l^{(n)}(\xi,\eta,\tau)e^{i(kx-wt)l},\\ v=\sum_{n=1}^{\infty}\epsilon^{n+1}\sum_{l=-\infty}^{\infty}v_l^{(n)}(\xi,\eta,\tau)e^{i(kx-wt)l},\\ \phi=\sum_{n=1}^{\infty}\epsilon^n\sum_{l=-\infty}^{\infty}\phi_l^{(n)}(\xi,\eta,\tau)e^{i(kx-wt)l},\\ n_e=1+\sum_{n=1}^{\infty}\epsilon^n\sum_{l=-\infty}^{\infty}n_{el}^{(n)}(\xi,\eta,\tau)e^{i(kx-wt)l}. \end{cases} \tag{4}$$

Substituting Eq. (4) into Eqs. (1) and (3), collecting the terms in different powers of ϵ, we obtain the following equations at the lower order of ϵ:

$$n_{e1}^{(1)}=\frac{4}{4+H_e^2k^2}\phi_1^{(1)},\qquad n_{i1}^{(1)}=\frac{k^2}{w^2}\phi_1^{(1)},\qquad u_1^{(1)}=\frac{k}{w}\phi_1^{(1)},\qquad v_1^{(1)}=\frac{-i}{w}\frac{\partial\phi_1^{(1)}}{\partial\eta}. \tag{5}$$

For the next order of the ϵ, we have equation with $n=1$, $l=0$:

$$n_{e0}^{(1)}=\phi_0^{(1)}=0,\qquad n_{i0}^{(1)}=\beta\phi_0^{(1)}=0,\qquad u_0^{(1)}=c\beta\phi_0^{(1)}=0,\qquad \frac{\partial v_0^{(1)}}{\partial\xi}=\frac{k^2\frac{\partial|\phi_1^{(1)}|^2}{\partial\eta}+w^2\frac{\partial\phi_0^{(2)}}{\partial\eta}}{cw^2}. \tag{6}$$

The dispersion relation $w^2=\frac{k^2(4+H_e^2k^2)}{4\beta+4k^2+k^4H_e^2}$ is also deduced. Therefore, the group velocity $c=\frac{\partial w}{\partial k}=\frac{8\beta(H_e^2k^2+2)}{(k^4H_e^2+4k^2+4\beta)^{\frac{3}{2}}(H_e^2k^2+4)^{\frac{1}{2}}}$, which is the compatibility condition obtained accordingly.