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Landauer's principle and non-equilibrium statistical ensembles

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Abstract

Landauer's principle is fundamental for the physics of information. It establishes the least amount of energy that needs to be dissipated in order to erase a bit of information. Using the Beck–Cohen representation of statistical ensemble distributions, we explore an extension of Landauer's principle to systems out of equilibrium.

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A few years ago, Beck and Cohen (BC) advanced an interesting treatment of non-equilibrium, meta-stable states [1–7]. In this approach statistical ensemble distributions are represented as a superposition of Gibbs distributions that are characterized by different temperatures. It is hence also referred to as "superstatistics" and it appears to be particularly successful in dealing with various physical settings, most notably turbulence [8–11]. In the present Letter we use the BC formalism to discuss a fundamental aspect of the physics of information: Landauer's principle.

The physics of information [12–20] has been receiving increasing attention [21–27]. There is a growing consensus that information is endowed with physical reality, not in the least because the ultimate limits of any real device that processes or transmits information are determined by the fundamental laws of physics [21–24,28,29]. At the same token a plenitude of theoretical developments rendered the concept of information an essential ingredient for a deep understanding of physical systems and processes [13–17,21]. Landauer's principle is one of the most basic results in the physics of information [19] and constituted a historical turning point in the field by directly connecting information processing with (more) conventional physical quantities [30]. Above all, it allowed for exorcising Maxwell's demon [21]. According to Landauer's principle a minimal amount of energy is required to be dissipated in order to erase a bit of information in a computing device working at temperature T. This minimum energy is given by $kT \ln 2$, when k denotes Boltzmann's constant [31–33]. Landauer's principle has profound implications as it allows for novel, physically motivated derivations of several important results in classical and quantum information theory [34]. In addition, it comprises a quite useful heuristic tool for establishing new links between, or obtaining new derivations of, fundamental aspects of thermodynamics and other areas of physics [35].

Most derivations of Landauer's principle can be considered semi-phenomenological since they are readily based on the second principle of thermodynamics. Alternative derivations, which build on dynamical principles, assume that the systems under study are in thermal equilibrium and can thus be described in terms of Gibb's canonical distributions. In view of the

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fundamental character of Landauer's principle, however, it is highly desirable to explore more general formulations applicable to non-equilibrium settings involving non-Gibbs ensemble distributions. Information processing can be realized in various physical settings. Indeed, one of the main ideas behind the physics of information and computation is that every physical system (even the whole universe) can be construed as an information processing system. Consequently, it is of considerable interest to extend the fundamental principles of the physics of information to out of equilibrium situations. This more general case may encompass compelling examples of physical realizations of information processes such as, for instance, those related to biology. Biological systems process information at molecular, cellular, and higher levels [36] within an essentially no-equilibrium setting. Indeed, information processing is clearly at the very foundations of biology and has been appropriately characterized as the "touchstone of life" [36]. Due to the extremely low concentrations of some important molecules involved in the aforementioned biological processes, stochastic fluctuations akin to the temperature fluctuations associated with the BC formalism play a fundamental role in biological scenarios [37] (the possible application of non-equilibrium metastable states described by non-Gibbsian ensembles to biochemical systems has been recently suggested in [38]).

As part of the program of exploring some basic aspects of the physics of information within non-equilibrium scenarios, we here extend Landauer's principle to systems out of equilibrium described by the BC formalism. We consider this particular problem a natural starting point because the BC approach, while useful for describing various non-equilibrium problems, still is to a large extent based upon the Gibbs canonical formalism.

We adopt the conventional picture for an erasure process [32] and assume that the composite system's Hamiltonian has the form $H = H^{(s)} + H^{(b)} + H^{(i)}$. Here $H^{(s)}$ denotes the Hamiltonian of the storage device for a bit of information. More specifically, $H^{(s)}$ describes a (classical) particle with mass m moving in a one-dimensional, symmetric double well potential V centered at origin. The left and right wells correspond to the 0 and 1 states of the stored bit, respectively (see Fig. 1). $H^{(b)}$ represents the Hamiltonian of a surrounding heat bath. Further, we incorporate a finite interaction term $H^{(i)}$, describing the interplay between the bit-storing device and the heat bath. This interaction term, however, is always considered much smaller than both $H^{(s)}$ and $H^{(b)}$. In general, $H^{(s)}$ and $H^{(i)}$ are time-dependent Hamiltonians. In particular, the shape of the aforementioned potential function V changes during the erasure process. Importantly, however, the Hamiltonians do not differ before and after erasure. That is, the shape of the potential function before erasure, $V(\dots; t = 0)$, matches the form after erasure has been completed. Prior to erasure the composite system is described by the ensemble distribution $\tilde{F}_{\text{initial}} = \tilde{F}_{\text{initial}}(\boldsymbol{x}^{(s)}, \boldsymbol{x}^{(b)})$, where $\boldsymbol{x}^{(s)}$ and $\boldsymbol{x}^{(b)}$ summarize the complete sets of canonical variables of the storage device s and the heat bath b, respectively. After erasure the composite system is given by the final distribution $\tilde{F}_{\text{final}} = \tilde{F}_{\text{final}}(\boldsymbol{x}^{(s)}, \boldsymbol{x}^{(b)})$. The corresponding marginal distributions read $\tilde{F}^{(s)}_{\text{initial/final}} = \tilde{F}^{(s)}_{\text{initial/final}}(\boldsymbol{x}^{(s)}) = \int \tilde{F}_{\text{initial/final}}\, d\Omega^{(b)}$ and $\tilde{F}^{(b)}_{\text{initial/final}} = \tilde{F}^{(b)}_{\text{initial/final}}(\boldsymbol{x}^{(b)}) = \int \tilde{F}_{\text{initial/final}}\, d\Omega^{(s)}$. The erasure process starts with an initial distribution that is associated with thermal equilibrium at temperature $T = (k\beta)^{-1}$, given by the Gibbs canonical distribution:

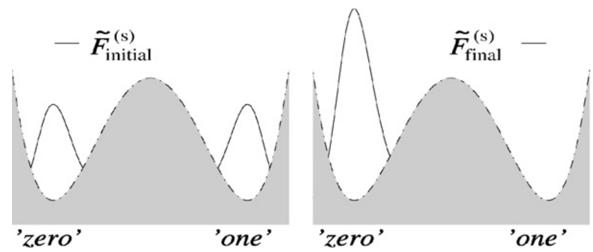

Fig. 1. Illustration of the erasure principle. In the initial state (left panel) both wells are equally filled, that is, there is an equal probability for the bit to be in the 'zero' or 'one' state. After erasure, in contrasts, well 'one' is empty and the distribution, i.e., all probability is concentrated in well 'zero' and the bit is re-set to state 'zero' (right panel); see text for further details.

$$\tilde{F}_{\text{initial}}\big(\boldsymbol{x}^{(s)}, \boldsymbol{x}^{(b)}; \beta\big) = \frac{e^{-\beta H_{\text{initial}}}}{Z(\beta)} \tag{1}$$

with H_{initial} denoting the system's total Hamiltonian at the initial time. $Z(\beta)$ is the accompanying partition function of the composite system given by

$$Z(\beta) = \int e^{-\beta H_{\text{initial}}}\, d\Omega. \tag{2}$$

To describe the bit-storing potential V, we denote the canonical variables as $\boldsymbol{x}^{(s)} = \{q, p\}$ and the corresponding volume element as $d\Omega^{(s)} = dq\, dp$. Then, the initial marginal distribution for the device reads

$$\tilde{F}^{(s)}_{\text{initial}} = \frac{e^{-\beta p^2/2m}}{Z^{(s)}(\beta)} e^{-\beta V(q;t=0)} \quad \text{for } |q| < \infty, \tag{3}$$

with

$$Z^{(s)}(\beta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\beta p^2/2m} e^{-\beta V(q;t=0)}\, dq\, dp. \tag{4}$$

After erasure, the composite system, i.e., storage device plus heat bath, has evolved into a new state whose final distribution $\tilde{F}_{\text{final}} = \tilde{F}_{\text{final}}(\boldsymbol{x}^{(s)}, \boldsymbol{x}^{(b)}; \beta)$ yields a marginal distribution of the storage device in the form of, e.g.,

$$\tilde{F}^{(s)}_{\text{final}} = \begin{cases} 2\tilde{F}^{(s)}_{\text{initial}} & \text{for } -\infty < q < 0, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The double well potential $V(q, t)$ and the projections of the probability distributions (1) and (3) onto the configuration space (q) are depicted in Fig. 1—notice that Fig. 1 represents the conventional model for the erasure of a bit of information in a memory storage device. The inverse temperature β describing the initial distribution can be seen as a parameter that characterizes a family of different realizations of the erasure process $\tilde{F}_{\text{initial}}(\boldsymbol{x}^{(s)}, \boldsymbol{x}^{(b)}; \beta) \to \tilde{F}_{\text{final}}(\boldsymbol{x}^{(s)}, \boldsymbol{x}^{(b)}; \beta)$. Each realization is associated with a particular time-dependent solution of Liouville's equation, $\frac{\partial F}{\partial t} = [F, H]$. Notice, however, that the total