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Synchronization defect lines in complex-oscillatory target waves

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Abstract

It is well known that one of key features of spiral waves in complex-oscillatory media is the appearance of synchronization defect lines, across which the phase of the oscillation changes by multiples of 2π. In this Letter, we report the appearance of synchronization defect lines in target waves in complex-oscillatory media by studying a model of two-dimensional Rössler reaction–diffusion system subject to an appropriate periodic force in a small region of the center of domain. The geometric structure and stability of the defect lines are studied.

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Keywords: Target waves; Synchronization defect lines; Spiral waves

Target waves (TWs) and spiral waves (SWs) are the two most commonly observed patterns in nature [1]. They appear in a large variety of spatio-temporal systems, such as chemical reaction–diffusion systems [2], dictyostelium discoideum systems [3], heart muscle systems [4], and so on. These two types of patterns have drawn a great deal of attention and have been extensively investigated in both excitable and oscillatory media. In oscillatory media, generic features of spiral wave dynamics are usually described in terms of the complex Ginzburg–Landau equation (CGLE), which is a normal form in the vicinity of a Hopf bifurcation to an oscillatory state in spatially extended systems. Recent studies found SWs can even exist in complex-oscillatory media where the local dynamics exhibits complex-oscillatory or even chaotic behavior [5–10]. In such parameter regimes, a novel phenomenon appears, which is characterized by a synchronization defect line (SDL) across which the phase of the oscillation changes by multiples of 2π. Very recently, Sandstede and Scheel [11] investigated period doubled spiral waves and line defects in considerable mathematical detail. SDLs have also been well observed in experiments mainly in the Belousov–Zhabotinsky (BZ) reaction under oscillatory conditions [12–15]. For recent review article on this subject, see Ref. [16]. On the other hand, TWs have also attracted much attention. For example, Hendrey et al. [17] observed two types of TW patterns (stationary and breathing) by introducing a spatially localized inhomogeneity into the CGLE, an effective method for controlling spiral turbulence by producing a TW was considered [18,19], Stich et al. [20] analyzed the properties of target patterns in heterogeneous oscillatory systems created by the pacemaker, and, very recently, Gao and Zhan [21] studied the possibility of producing a stable target wave by simply setting a constant system variable in a small spatial region, i.e., the so-called system variable block method. To the best knowledge of the authors, in contrast to the well-known SWs in complex-oscillatory media, TW patterns in complex-oscillatory media, however, have never been studied or documented in the literature. In particular, some relevant problems remain unclear, such as how to produce complex-oscillatory TWs, is it possible to generate SDLs in space, and what are the essential features of SDLs. In this Letter we will address these issues by investigating a model of two-dimensional Rössler reaction–diffusion system and focus on some unusual features of synchronization defect lines of TWs in complex-oscillatory media, such as the geometrical structure and dynamical stability.

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Consider the reaction–diffusion system

$$\frac{\partial \mathbf{c}(\mathbf{r},t)}{\partial t} = \mathbf{R}\big[\mathbf{c}(\mathbf{r},t)\big] + D\nabla^2\mathbf{c}(\mathbf{r},t), \tag{1}$$

where $\mathbf{c}(\mathbf{r},t)$ is a vector of time-dependent concentrations at point $\mathbf{r}$ in the two-dimensional domain of a square $L \times L$ or a disk with radius R. Without loss of generality, $L = 128$ and $R = 62.5$ are considered in the Letter. Larger L and R were also tested and we found that they do not change the qualitative results. The diffusion coefficient D for all three species are taken to be the same: $D = 0.1$. No-flux boundary conditions are considered. In numerics, the explicit Euler method was employed by setting the space and time steps: $\Delta x = 0.5$ and $\Delta t = 0.01$. $\mathbf{R}[\mathbf{c}(\mathbf{r},t)]$ describes the local reaction kinetics and in the Letter is specified by the Rössler model: $R_x = -c_y - c_z$, $R_y = c_x + 0.2c_y$, and $R_z = c_x c_z - Cc_z + 0.2$. We will change parameter C within (2.0, 6.0). As C is increased, the single Rössler oscillator undergoes a period-doubling route to chaos; the first two period-doubling bifurcations take place at $C \simeq 2.83$ and $C \simeq 3.86$ for the transitions from $P1$ to $P2$ and from $P2$ to $P4$, respectively. For the appearance of SWs in complex oscillatory media, studies have demonstrated that there are slightly small parameter lags and the two corresponding bifurcations in patterns take place at $C \simeq 3.07$ and $C \simeq 4.075$, respectively [5–7].

For spiral waves in P1 media (or the so-called simple spiral wave), the phase field $\phi(\mathbf{r},t)$ contains a topological defect point in the spiral core such that the integral of the phase field gradient taken along any contour encircling the core takes a nonzero value,

$$\frac{1}{2\pi}\oint \nabla\phi(\mathbf{r},t)\cdot d\mathbf{l} = \sigma. \tag{2}$$

The integer σ is the topological charge of the defect and $\sigma \neq 0$. For spiral waves in complex-oscillatory media (or the so-called complex-oscillatory spiral), this restriction in the above equation ($\sigma \neq 0$) persists. In such complex-oscillatory parameter regimes, synchronization defect lines spontaneously appear, which separate domains of different oscillation phases and across which the phase changes by multiples of 2π. The underlying mechanism now is clear [5] and such SDLs are believed to arise from the need to reconcile the rotation period of a one-armed spiral wave with the oscillation period of the local dynamics. In simple model studies, SDLs appear as stationary and a slightly curved line, which connects central core region and boundary. Recently, Zhan and Kapral [10] well proposed a theoretical model to reconstruct the geometrical structure of such SDLs. In chemical BZ reactions, however, the structures of SDLs can be much rich. In particular, Park and Lee [13] found stationary SDLs can show spiral shapes and the tip of SDLs in the core region may even execute a meandering motion like that seen in excitable media. If three-dimensional effects are considered, much richer and complicated dynamics of SDLs are possible [15]. In sharp contrast to the spiral waves, target waves radiate concentric circular waves and are devoid of any defect point ($\sigma = 0$). This restriction may persist for complex-oscillatory TWs and give rise to some novel phenomena, as we will see below.

Similar to the approach to producing TWs in CGLE used in Ref. [18], we apply an external periodic signal to a fixed small area at the center of the domain, i.e., we add to the right-hand of Eq. (1) the term $\Gamma\delta_{i,\mu}\delta_{j,\nu}\sin(\omega t)$, where i and j are the integer numbers corresponding to whole discretized x and y spatial variables, whereas μ and ν are the integer numbers corresponding to the stimulus region. Γ and ω are the driving strength and driving frequency, respectively. Note that this driving force is added to all three Rössler variable terms. The small controlled area is taken to be a disk with radius $r = 7.5$. Compared with the radius R ($R = 62.5$) of the whole domain, this size is very small, but this force might dramatically change the system behavior under certain conditions. We have well studied the conditions for the appearance of target waves by choosing an appropriate diving strength Γ ($\Gamma = 0.2$) and searching a wide range of the values of ω. When the stimulus frequency is lower than about 3, i.e., $\omega < 3$, TWs can be produced. We also found that the values of parameter C for the establishment of TWs are the same as those for SWs presented earlier, namely, $C \simeq 3.07$ and $C \simeq 4.075$ corresponding to the transitions from P1 to P2 and from P2 to P4, respectively.

The appearance of SDLs heavily relies on the choice of the external periodic signals (namely, the values of Γ and ω), and the stimulus frequency ω may play a more significant role, compared with Γ. We found that under the P2 oscillatory condition, if the stimulus frequency is near one or two times of the bulk oscillatory frequency $\omega_o \simeq 0.534$, SDLs can be easily observed; in the P4 oscillatory media, the range of the stimulus frequency that is capable of generating SDLs is larger, and the ω should be around integer time (1, 2, 3, or 4) of the P4 oscillatory frequency $\omega_o \simeq 0.267$. In the following discussion, the stimulus frequency is chosen to be fixed $\omega = 1.068$. An interesting finding is that initial conditions may also play an important role; for example, homogeneous initial conditions usually cannot produce SDLs, but random initial conditions that lead to inhomogeneous phases can.

Next we pay our attention to the geometrical structure of SDLs in complex-oscillatory TWs. As $C \in [3.07, 4.075]$ within the P2 regime, under suitable setting of internal periodic signals and random conditions, fine and stationary SDLs can be observed. As shown in Fig. 1, the SDLs in the patterns of TWs in the square and disks are visible. $C = 3.5$, $\Gamma = 0.2$, and $\omega = 1.068$. They appear as a clamp structure, constructed by two straight lines, whose free ends are pinned to the boundaries, and one small bending part, which connects the two straight lines. In the left panel of the figure, we show the target patterns by the value of the $c_x(\mathbf{r},t)$ concentration field at one time instant. In the right panel of the figure, we illustrate the spatial structure of SDLs for the corresponding target pattern by calculating the scalar fields $\Delta c_x(\mathbf{r},t)$. $\Delta c_x(\mathbf{r},t) = \frac{1}{\tau}\int_0^\tau |c_x(\mathbf{r},t+t') - c_x(\mathbf{r},t+t'+\tau)|\,dt'$, for $\tau = T/2$, where T is the period of the period-2 target wave ($T = 11.76$). $c(\mathbf{r},t) = c(\mathbf{r},t+T/2)$ for the P1 points on the line defect, while this relation does not apply for other spatial points with P2 dynamics. Thus, this time-average method can be easily implemented to locate the line defects in the media [13], and has already been extensively used in the studies of complex-oscillatory SWs.