Available online at www.sciencedirect.com
ScienceDirect
Physics Letters A 372 (2008) 2464–2473
PHYSICS LETTERS A
www.elsevier.com/locate/pla

Fully frustrated Josephson junction ladders with Mobius boundary conditions as topologically protected qubits

Gerardo Cristofano [a], Vincenzo Marotta [a], Adele Naddeo [b,*], Giuliano Niccoli [c]

[a] *Dipartimento di Scienze Fisiche, Universitá di Napoli "Federico II" and INFN, Sezione di Napoli, Via Cintia, Complesso Universitario M. Sant'Angelo, 80126 Napoli, Italy*

[b] *Dipartimento di Fisica "E.R. Caianiello", Universitá degli Studi di Salerno and CNISM, Unità di Ricerca di Salerno, Via Salvador Allende, 84081 Baronissi (SA), Italy*

[c] *LPTM, Université de Cergy-Pontoise, 2 avenue Adolphe Chauvin, 95302 Cergy-Pontoise, France*

Received 5 July 2007; received in revised form 25 October 2007; accepted 26 November 2007

Available online 5 December 2007

Communicated by R. Wu

Abstract

We show how to realize a "protected" qubit by using a fully frustrated Josephson junction ladder (JJL) with Mobius boundary conditions. Such a system has been recently studied within a twisted conformal field theory (CFT) approach [G. Cristofano, G. Maiella, V. Marotta, Mod. Phys. Lett. A 15 (2000) 1679; G. Cristofano, G. Maiella, V. Marotta, G. Niccoli, Nucl. Phys. B 641 (2002) 547] and shown to develop the phenomenon of flux fractionalization [G. Cristofano, V. Marotta, A. Naddeo, G. Niccoli, Eur. Phys. J. B 49 (2006) 83]. The relevance of a "closed" geometry has been fully exploited in relating the topological properties of the ground state of the system to the presence of half flux quanta and the emergence of a topological order has been predicted [G. Cristofano, V. Marotta, A. Naddeo, J. Stat. Mech.: Theory Exp. (2005) P03006]. In this Letter the stability and transformation properties of the ground states under adiabatic magnetic flux change are analyzed and the deep consequences on the realization of a solid state qubit, protected from decoherence, are presented.

PACS: 11.25.Hf; 03.75.Lm; 74.81.Fa

Keywords: Fully frustrated Josephson junction ladder; Topological order; Qubit

Arrays of weakly coupled Josephson junctions provide an experimental realization of the two-dimensional ($2D$) XY model. A Josephson junction ladder (JJL) is the simplest quasi-one-dimensional version of an array in a magnetic field [1]; recently such a system has been the subject of many investigations because of its possibility to display different transitions as a function of the magnetic field, temperature, disorder, quantum fluctuations and dissipation. In a recent paper [2] we analyzed the phenomenon of fractionalization of the flux quantum $\frac{hc}{2e}$ in a fully frustrated JJL in order to investigate how the phenomenon of Cooper pair condensation could cope with properties of charge (flux) fractionalization, typical of a low-dimensional system with a discrete Z_2 symmetry. The role of such a symmetry was recognized to be crucial for demanding more general boundary conditions, of the Mobius type, at the end sites of the ladder [2]. The same feature was evidenced also in quantum Hall systems in the presence of impurities or defects [3–5]. Furthermore a Z_2 symmetry is present in the fully frustrated XY (FFXY) model or equivalently, see Refs. [6,7], in two-dimensional Josephson junction arrays (JJA) with half flux quantum $\frac{1}{2}\frac{hc}{2e}$ threading each square cell and accounts for the degeneracy of the ground state. We noticed how it was possible to generate non-trivial topologies, i.e. the torus, in the context of a CFT approach, which allowed us to construct a ground state wave function, whose center of charge could describe a coherent superposition of localized states sharing all the non-

* Corresponding author.
E-mail address: naddeo@sa.infn.it (A. Naddeo).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2007.11.048

trivial global properties of the order parameter. In particular for the FFXY model they were shown to be closely related to the presence of half flux quanta, also viewed as topological defects [2]. The emergence of topological order in fully frustrated JJLs with non-trivial geometry has been predicted and fully exploited in Ref. [8] by means of CFT techniques. Such a concept was first introduced in order to describe the ground state of a quantum Hall fluid [9] but today it is of much more general interest [10]. Two features of topological order are very striking: fractionally charged quasiparticles and a ground state degeneracy depending on the topology of the underlying manifold, which is lifted by quasiparticles tunneling processes. In general a system is in a topological phase if its low-energy, long-distance effective field theory is a topological quantum field theory that is, if all of its physical correlation functions are topologically invariant up to corrections of the form $e^{-\frac{\Delta}{T}}$ at temperature T for some non-zero energy gap Δ. More recently superconductors have been proposed in which superconductivity arises from a topological mechanism rather than from a Ginzburg–Landau paradigm: the key feature is a mapping on an effective Chern–Simons gauge theory, which turns out to be exact in the case of JJA and frustrated JJA [11]. As we will stress in the following, topological order is crucial for the implementation of fully frustrated JJLs as "protected" qubits [12,13] in solid state quantum computation realm. The idea in all such realizations is that the systems involved (large and small size Josephson junction arrays of special geometry [12,14]) share the property that, in the classical limit for the local superconducting variables, the ground state is highly degenerate. The residual quantum processes within such a low energy subspace lift the classical degeneracy in favor of macroscopic coherent superpositions of classical ground states [14]. An example of such a system has been proposed, which consists of chains of rhombi frustrated by an half flux quantum [14] with the property that in the classical limit each rhombus has two degenerate states. The protected degeneracy in all such systems emerges as a natural property of the lattice Chern–Simons gauge theories which describe them [14]. In general, if a physical system has topological degrees of freedom that are insensitive to local perturbations (that is noise), then information contained in those degrees of freedom would be automatically protected against errors caused by local interactions with the environment [13].

The aim of this Letter is to show how to realize a "protected" qubit in terms of a fully frustrated Josephson Junction ladder (JJL) with Mobius boundary conditions by fully exploiting the implications of "closed" geometries on the ground state global properties of the system, already studied in Ref. [2]. Such a qubit would be the elementary building block of a "protected" quantum computer. The task appears to be not very simple; in general we need a quantum system with 2^K quantum states (K being the number of big openings in the Josephson system under study) which are degenerate in the absence of external perturbations and are robust against local random fluctuations, that is against noise. This means that any coupling to the environment does not induce transitions between the 2^K quantum states or change their relative phases. Summarizing, we need a system, whose Hilbert space contains a 2^K-dimensional subspace characterized by the crucial property that any local operator $\hat{O}$ has only state-independent diagonal matrix elements up to vanishingly small corrections: $\langle n|\hat{O}|m\rangle = O_0\delta_{mn} + o[\exp(-L)]$, L being the system size. A possible answer to such a highly non-trivial requirement could be a system with a protected subspace built up by a topological degeneracy of the ground state [13]. An alternative approach would be to exhibit a low-energy effective field theory for the system under study which is a topological one and whose vacua are topologically degenerate and, then, robust against noise. This is the approach which we follow in the present Letter; in particular we show how to get a protected subspace with 2^K quantum states, $K = 1$, by considering a Josephson junction ladder and closing it by imposing Mobius boundary conditions, in order to get a non-trivial topology. We will show that such a system is described by a low-energy effective field theory which is a twisted conformal field theory [3,4,15,16]. Such a theory accounts very well for the topological properties of the system under study [2,8]. In particular we analyze the stability and transformation properties of the ground state wave functions under adiabatic magnetic flux change; in this way we are able to identify the two states of a possible protected qubit and also to describe its manipulation: "flip state" processes.

We recall that Josephson junction arrays (JJA) are a very useful tool for investigating quantum-mechanical behaviour in a wide range of parameters space, from $E_C \gg E_J$ (where $E_C = \frac{(2e)^2}{C}$ is the charging energy and $E_J = \frac{\hbar}{2e} I_c$ is the Josephson coupling energy; C is the capacitance of each island and I_c is the critical current of each junction) to $E_J \gg E_C$. In fact there exists a couple of conjugate quantum variables, the charge and phase of each superconducting island, and two dual descriptions of the array can be given [17]: (a) through the charges (Cooper pairs) hopping between the islands, (b) through the vortices hopping between the plaquettes. Furthermore in the presence of an external magnetic field charges gain additional Aharonov–Bohm phases and, conversely, vortices moving around islands gain phases proportional to the average charges on the islands [18]. Such basic quantum interference effects found applications in recent proposals for solid state qubits for quantum computing, based on charge [19] or phase [20] degrees of freedom in JJAs. "Charge" devices operate in the regime $E_C \gg E_J$ while "phase" or "flux" devices are characterized by strongly coupled junctions with $E_J \gg E_C$.

Let us now focus on the simplest physical array one can devise in order to meet all the above requests, that is a Josephson junction ladder with N plaquettes closed in a ring geometry with a half flux quantum ($\frac{1}{2}\Phi_0 = \frac{1}{2}\frac{hc}{2e}$) threading each plaquette [1], and describe briefly its general properties before introducing an interaction of the charges (Cooper pairs) with a magnetic impurity (defect), as drawn in Fig. 1. With each site i we associate a phase φ_i and a charge $q_i = 2en_i$, representing a superconducting grain coupled to its neighbors by Josephson couplings; n_i and φ_i are conjugate variables satisfying the usual phase-number commutation relation. The Hamiltonian describ-