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Adaptive–impulsive synchronization of uncertain complex dynamical networks

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Abstract

This Letter studies adaptive–impulsive synchronization of uncertain complex dynamical networks. Based on the stability analysis of impulsive system, several network synchronization criteria for local and global adaptive–impulsive synchronization are established. Numerical example is also given to illustrate the results.

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1. Introduction

Recently, complex dynamical networks have attracted a great deal of attentions. One of the interesting phenomena in complex dynamical networks is the synchronization of all dynamical nodes in the network [1–9]. Most of the existing works on synchronization of complex networks consider a given network, that is, the dynamical of each node and coupling functions are known [2–5]. However, in practice, it is often difficult to get the exact estimation of the coupling strength. And we have little information about the network structure. On the other hand, it is also noticed that in these schemes the final state of the network, where the network will reach after achieving synchronization, is usually not known in advance and cannot be changed as design [8]. Therefore, it would be interesting to study a general situation where the network coupling is unknown but bounded linear/nonlinear function, and the network structure is only partially known or completely unknown a priori. And it is desirable if the state that the network will synchronize to can be chosen.

In this Letter, we study the synchronization of uncertain dynamical networks, and propose an adaptive–impulsive synchronization approach for uncertain complex dynamical networks. Based on the comparison theorem for the stability of impulsive control system developed in [11], several network synchronization criteria are established. We design robust adaptive–impulsive controllers which can ensure that the states of the dynamical network locally or globally asymptotically synchronize with an arbitrarily assigned state of an isolate node of the network. In comparison with the results of the adaptive-feedback approach [9,10], the proposed scheme has faster convergence rate and smaller final coupling strength. In addition, network could be synchronized in larger impulsive distance compare to ordinary impulsive synchronization schemes [6,7].

This Letter is organized as follows. In Section 2, we first describe the general uncertain complex dynamical model and formulate the problem of adaptive–impulsive synchronization for an uncertain dynamical network. In Section 3, local and global

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adaptive–impulsive synchronization criteria are established. In Section 4, numerical example is given to demonstrate the effectiveness of the proposed controller design methods. Finally, conclusions are given in Section 5.

2. Formulation of the problem

This section introduces an uncertain complex dynamical network model and formulates the adaptive–impulsive synchronization scheme for the dynamical network.

Consider an uncertain complex dynamical network consisting of N identical nonlinear oscillators with uncertain nonlinear diffusive couplings, which is described by

$$\dot{x}_i = f(x_i, t) + h_i(x_1, x_2, \dots, x_N), \quad t \geqslant 0 \tag{1}$$

for $i = 1, 2, \dots, N$, where each node is an n-dimensional dynamical system with node dynamics $\dot{x} = f(x, t)$, $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in R^n$ is the state variable of node i, $f : R_+ \times R^n \to R^n$ is a smooth nonlinear vector field, $h_i : R^m \to R^n$ are unknown nonlinear smooth diffusive coupling functions, where $m = nN$.

Defined that the network (1) is synchronized if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t), \quad \text{as } t \to \infty, \tag{2}$$

where $s(t)$ is a solution of an isolate node of the network, i.e.,

$$\dot{s}(t) = f\big(s(t), t\big) \tag{3}$$

existing for all $t \in R_+$. Here, $s(t)$ can be an equilibrium point, a periodic orbit, an aperiodic orbit, or a chaotic orbit in the phase space.

Define the synchronization error as $e_i(t) = x_i(t) - s(t)$. Then the objective of this Letter is to design an adaptive-feedback controller u_i and an impulsive controller $U_{i_k}(x_i - s, t)$ such that the state of dynamical network (1) synchronizes with the state of node (3). That is

$$\lim_{t \to \infty} \big\| e_i(t) \big\| = 0. \tag{4}$$

Under the adaptive–impulsive control, the controlled state of network (1) satisfies the following system:

$$\begin{cases} \dot{x}_i = f(x_i, t) + h_i(x_1, x_2, \dots, x_N) + u_i, & t \neq \tau_k, \\ \Delta x_i = x_i(\tau_k^+) - x_i(\tau_k^-) = U_{i_k}(x_i - s, t), \\ \quad t = \tau_k, \ k = 1, 2, \dots, \end{cases} \tag{5}$$

where $i = 1, 2, \dots, N$, $x_i(\tau_k^+)$ is the right limit of $x_i(\tau_k)$ at $t = \tau_k$, and $x_i(\tau_k^-)$ is the left limit, $\{\tau_k\}$ satisfies $0 \leqslant \tau_1 < \tau_2 < \cdots$, which denotes the times when impulsive jumps occur. In this system, there are two kinds of control inputs. The first kind is the continuous control inputs u_i and the second one is the impulsive control inputs U_{i_k}. It is assumed that when the network achieves synchronization as defined in (2), the coupling terms should vanish: i.e., $h_i(s, s, \dots, s) + u_i = 0$.

Then the error dynamical system is given by

$$\begin{cases} \dot{e}_i = \tilde{f}(x_i, s, t) + \tilde{h}_i(x, s) + u_i, & t \neq \tau_k, \\ \Delta e_i = u_{i_k}(t, e_i), & t = \tau_k, \ k = 1, 2, \dots, \end{cases} \tag{6}$$

where $\tilde{f}(x_i, s, t) = f(x_i, t) - f(s, t)$, and $\tilde{h}_i(x, s) = h_i(x_1, x_2, \dots, x_N) - h_i(s, s, \dots, s)$.

Hypotheses 1. (H1) Suppose that there exist nonnegative constants γ_{ij} $(1 \leqslant i, j \leqslant N)$ satisfying

$$\big\| h_i(x_1, x_2, \dots, x_N) - h_i(s, s, \dots, s) \big\| \leqslant \sum_{j=1}^{N} \gamma_{ij} \| e_j \|. \tag{7}$$

The continuous control inputs u_i here are adaptive-feedback controller defined as

$$u_i = -d_i e_i, \quad i = 1, 2, \dots, N \tag{8}$$

and updating laws

$$\dot{d}_i = k_i e_i^T e_i = k_i \| e_i \|^2, \quad i = 1, 2, \dots, N, \tag{9}$$

where $k_i > 0$ are arbitrary constants. Thus, instead of the usual linear feedback, the feedback strength d_i will be adapted with the synchronization error. And if the network could be synchronized ($e \to 0$), d_i would get to an ultimate feedback strength $\hat{d}_i$.

In the following, we choose a linear impulsive controller, i.e., $U_{i_k}(x_i - s, \tau_k) = B_{i_k}(x_i - s, \tau_k)$, where each B_{i_k} is an $n \times n$ constant matrix.

3. Adaptive–impulsive synchronization of an uncertain dynamical network

This section discusses the local synchronization and global synchronization of the uncertain dynamical network (1).

3.1. Local synchronization

Let $A(t) = Df(s, t)$ denote the Jacobian of f on $s(t)$. Then linearizing error system (6) around zero and applying the adaptive controller (8) and (9) to it gives

$$\begin{cases} \dot{e}_i = A(t) e_i + \tilde{h}_i(x, s) - d_i e_i, & t \neq \tau_k, \\ \Delta e_i = B_{ik} e_i, & t = \tau_k, \\ e_i(t_0^+) = e_{i0}, & i = 1, 2, \dots, N, \ k = 1, 2, \dots, \\ \dot{d}_i = k_i \| e_i \|^2. \end{cases} \tag{10}$$

Hypotheses 2. (H2) Assume that there exists a nonnegative constant α satisfying $\| Df(s, t) \| = \| A(t) \| \leqslant \alpha$.

If (H2) holds, then we get $\| (A(t) + A^T(t))/2 \| \leqslant \alpha$. Based on (H1) and (H2), a network synchronization criterion is deduced as follows.

Theorem 1. *Suppose that* (H1) *and* (H2) *hold, the initial feedback strength* $d_{i0} > 0$ $(1 \leqslant i \leqslant N)$ *is limited such that*

$$\delta_k = \lambda_{\max}\big(\Gamma + \operatorname{diag}\{\alpha - d_{10}, \alpha - d_{20}, \dots, \alpha - d_{N0}\}\big) \geqslant 0, \tag{11}$$

then the uncertain network (1) *is locally robust asymptotically synchronized with node* (3) *if the following conditions hold.*

$$(1) \quad \Delta_k = \sup_k \{\tau_{k+1} - \tau_k\} < \infty, \tag{12}$$

$$(2) \quad \lambda_{\max}\big((I + B_{i_k})^T (I + B_{i_k})\big) = \rho_k < 1. \tag{13}$$