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Effects of time-periodic linear coupling on two-component Bose–Einstein condensates in two dimensions

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Abstract

We examine two-component Gross–Pitaevskii equations with nonlinear and linear couplings, assuming self-attraction in one species and self-repulsion in the other, while the nonlinear inter-species coupling is also repulsive. For initial states with the condensate placed in the self-attractive component, a sufficiently strong linear coupling switches the collapse into decay (in the free space). Setting the linear-coupling coefficient to be time-periodic (alternating between positive and negative values, with zero mean value) can make localized states quasi-stable for the parameter ranges considered herein, but they slowly decay. The 2D states can then be completely stabilized by a weak trapping potential. In the case of the high-frequency modulation of the coupling constant, averaged equations are derived, which demonstrate good agreement with numerical solutions of the full equations.

Keywords: Nonlinear Schrödinger equations; Multiple components; Linear coupling

1. Introduction

Studies of matter-wave patterns in Bose–Einstein condensates (BECs), and especially solitons, have drawn a great deal of interest from experimentalists [1] and theorists [2,3] alike. In effectively one-dimensional (1D) traps ("cigar-shaped" ones), solitons have been created in condensates of ^{7}Li and ^{85}Rb atoms [4]. The attractive interaction between atoms, necessary for this, can be provided even in condensates with naturally repulsive interactions by switching the interaction type, with the help of the Feshbach resonance (FR) through an external magnetic field [5]. In the repulsive condensates, solitons of the gap type, supported by the periodic potential induced by optical lattices (OLs), have been predicted [6] and then demonstrated experimentally in the 1D setting (in the condensate of ^{87}Rb atoms) [7].

The creation of matter-wave solitons in 2D and 3D settings has not been reported yet, the most fundamental difficulty being the possibility of collapse in the same geometry [8], which makes the multidimensional solitons unstable. Various ways to stabilize 2D and 3D solitons against the collapse have been elaborated theoretically. One of them is the use of OLs, whose dimension may be equal to that of the underlying space [9], or smaller than it by 1 [10]. Another method relies on periodic time modulation of the effective strength of the nonlinear interaction, through the FR induced by an ac magnetic field. For the first time, a similar possibility was explored in a model of the propagation of $(2+1)$D spatial optical solitons (cylindrical beams) shone through a layered medium, which is built of periodically alternating layers with positive and negative values of the Kerr coefficient [11] (recently, the stabilization of spatial solitons of this type was demonstrated experimentally in an optical medium composed of

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alternating nonlinear and linear layers [12]). Directly in terms of the 2D BEC, the possibility to stabilize solitons by means of this *FR-management* technique, with the nonlinearity coefficient periodically alternating in time between positive and negative values, was predicted and analyzed in Refs. [13,14] (the same approach does not provide for the stabilization of 3D solitons, but it can do so in a combination with the quasi-1D OL potential [15]; the method does not stabilize 2D vortex solitons either [11]). In all these cases, it was found that the stabilization of the 2D solitons requires the time-average value of the nonlinearity coefficient to be different from zero, corresponding to the self-attraction (or self-focusing, in the optical medium). Still, it may happen that this stabilization mechanism generates an extremely long-lived transient dynamical regime, rather than a truly stable one: super-long simulations (on the time scale several orders of magnitude larger than the duration relevant to any experiment) reveal the beginning of what may be a very slow decay of the soliton through emission of radiation [16].

In the 1D geometry, the same FR-management technique gives rise to specific dynamical states of the condensate trapped in the static parabolic potential, such as breathers oscillating between Thomas–Fermi and quasi-soliton configurations, and stable two-soliton bound states [17]. Other states, predicted as a result of the interplay between the low-frequency modulation of the nonlinearity coefficient and the OL potential (in the 1D and 2D geometries alike) are *alternate solitons*, which periodically switch between gap-soliton and ordinary solitonic shapes [18]. It is relevant to mention that the FR-management technique belongs to the class of *soliton-management* methods, which were developed in nonlinear optics and then applied to BEC [19].

The effective stabilization of 2D solitons by means of the FR-management technique suggests another possibility: as is known, atoms of a given element (in particular, ^{7}Li) may exist in different intrinsic states, some of which interact repulsively between themselves, while others—attractively, the interaction between atoms belonging to the different states being repulsive in any case [4]; then, an external electromagnetic wave, resonantly coupling such two states, may give rise to oscillations between numbers of atoms in the self-attractive and self-repulsive states, and thus, possibly, help to stabilize 2D solitons against the collapse [20]. The objective of the present Letter is to report the first results of the theoretical analysis of the 2D model of this type, which may help to direct further studies of the topic. When the linear-coupling coefficient is subject to periodic modulation in time (in fact, periodic jumps between positive and negative values, with zero mean value), we conclude that the 2D soliton can be made quite long-lived, in the free space. If, in addition, a weak external trapping potential is added, the waveform features complete stabilization.

The above-mentioned resonant electromagnetic wave induces linear coupling between the wave functions of the two species (atomic states) in the respective system of coupled Gross–Pitaevskii equations (GPEs). A similar linear coupling (accounting for the interconversion between two states) was investigated in a number of settings (usually, assuming that the coupled species represent two different spin states of the same atom, while the electromagnetic wave is the spin-flipping one). Effects and patterns predicted in the framework of models with this type of the linear coupling include Josephson-like oscillations [21], domain walls [22], "breathe-together" oscillation modes [23], non-topological vortices [24], and a shift of the miscibility transition in binary Bose–Bose [25] and Fermi–Fermi [26] gases. However, to the best of our knowledge, effects of time-modulated ("managed") linear coupling have not been considered before, in this context.

The rest of the Letter is organized as follows. In Section 2, we introduce the model. In Section 3, its simplified version is derived, by means of the averaging method (assuming high-frequency time modulation of the linear-coupling coefficient). Numerical results, which support the basic inferences outlined above, are presented in Section 4, and the Letter is concluded by Section 5.

2. The model

The model is based on a pair of coupled 2D GPEs for mean-field wave functions of the two species, u and v. The general form of these equations is well known [27]. In the standard scaled form, it is

$$\begin{aligned} iu_t &= -(1/2)\nabla^2 u + \left(g_1|u|^2 + |v|^2\right)u + \epsilon^{-1}\kappa(t/\epsilon)v + (1/2)\Omega^2 r^2 u, \\ iv_t &= -(1/2)\nabla^2 v + \left(g_2|v|^2 + |u|^2\right)v + \epsilon^{-1}\kappa(t/\epsilon)u + (1/2)\Omega^2 r^2 v, \end{aligned} \tag{1}$$

where kinetic-energy operator $(1/2)\nabla^2$ acts on spatial coordinates x and y, Ω^2 is the strength of the isotropic trapping potential ($r^2 \equiv x^2 + y^2$), the coefficient accounting for the nonlinear repulsion between the species is normalized to be 1, and, in accordance with what said above, we assume the attraction between the atoms in the first species, and repulsion in the second, i.e., $g_1 < 0$ and $g_2 > 0$. Further, the form of the time-dependent linear-coupling coefficient, $\epsilon^{-1}\kappa(t/\epsilon)$, assumes a possibility of the high-frequency modulation corresponding to small ϵ (see below), while the function $\kappa(\tau)$ is chosen in the following form ($\tau \equiv t/\epsilon$),

$$\kappa(\tau) = \begin{cases} M, & 0 < \operatorname{mod}(\tau, T) < T/2, \\ -M, & T/2 < \operatorname{mod}(\tau, T) < T, \end{cases} \tag{2}$$

with period T.

Throughout this Letter, we fix the nonlinearity coefficients in Eqs. (1) to be $g_1 = -1$ and $g_2 = 1.03$. The proximity of the intra-species repulsive coefficient (g_2) to the one accounting for the inter-species repulsion (recall it was normalized to be 1) corresponds to the usual physical situation for ^{87}Rb, see, e.g., Ref. [28]. As for the intra-species attraction coefficient g_1 (which in ^{87}Rb could be produced by a Feshbach resonance), the choice of $g_1 = -1$ is a natural one, as we expect that the self-attraction and self-repulsion